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Botany. — "*Concerning the Influence of Light and Gravitation on Pellia epiphylla.*" By Dr. TH. WEEVERS. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of April 30, 1921.)

Recent investigations concerning the influence of light and gravitation on the growth and the curvature of plants have been undertaken only with some particular objects. Although this yields the advantage that such investigations may be complemental to each other, it yet seemed to me that the study of quite another object might open up new points of view, or might give rise to new questions.

I considered Pellia epiphylla Corda to be a suitable object, as it offered various advantages, notably for the study of the growth of sporogonia under the influence of stimuli of light and gravitation. To my knowledge nothing has been published on this point ever since 1874, when ASKENASY[1]) dealt with it in a few paragraphs.

After the cell-division has taken place, the sporogonia of Pellia come to a period of rest in winter. At its close the rising temperature brings about a growth of the stem-cells, slow at the beginning, afterwards more rapid; this process does not require illumination, an increased content of water of the soil accelerates it. The sporogonium-stem begins to grow within 3—5 days, only the base remains enclosed by the calyptra. The length of the stem increases from 2 or 3 mm. to 6 or 8 cm. The number of cells remains unchanged, so does the diameter approximately, but their length is increased from 25—30 μ to 800—900 μ, their volume becomes about 30 times as large again. The inner cells are torn up and the stem becomes hollow, finally dextro-torsion ensues. The starch, which fills up the cells completely during the winter-rest, is entirely lost during the growth of the cells; the chlorophyl-granules also disappear more and more, so that the green colour of the tissue in the fullgrown parts becomes transparent; only the oily bodies[2]) or elaioplasts[3]) in the outer cells remain unchanged.

The osmotic pressure, which prior to the longitudinal growth is

[1]) ASKENASY 1874. Botan. Ztg. 32.

[2]) W. PFEFFER 1874, Die Oelkörper der Lebermoose Flora.

[3]) J. H. WAKKER, Pringsh. Jahrb. Bd. 19.

extremely high towards the end of March, about 63 atmospheres (determined through plasmolysis with 17 % KNO_3 (weight mol.), isotonic coefficient 1.69[1])) diminishes gradually, with the increase of volume, to ± 7 atm. in the fully developed cells. The length of the cells used for our experiments varies from 15 to 25 mm.; the osmotic pressure from 13 to 16 atm.

The starch, when being consumed for the formation of cell-walls, is converted into sugars; a sugar (probably glucose) reducing a FEHLING's solution, was demonstrable in the young, in the growing cells and to a smaller quantity also in the fully developed cells.

This growth of cells does not occur simultaneously in all places; it begins and ends first in the middle parts; in the apical and especially in the basal part, enclosed by the calyptra, the growth continues longest; finally the length of the cells is nearly the same everywhere. We subjoin an instance (the measurements were marked with Indian ink).

24 March	at	8	a.m.	1	1	2	2	2	2	Total	10	m.m.
25	,,	,,	,,	3	3	3	3	3½	3	3	4	,,	25½	,,
26	,,	,,	,,	8	6	5	5	5	5	5	6	,,	45	,,
27	,,	,,	,,	12	7	6	5½	6	5½	6	8	,,	56	,,
28	,,	,,	,,	16	7	6	6	6	6	6	9	,,	62	,,

These values point to a not markedly pronounced large period in the total growth; the figures expressing the growth per hour also depend, however, on temperature and light. The slightly higher day-temperature (= 11°—14° C., room facing the North) accelerated the growth. Calculated per 12 hrs the successive values for another object are:

1st day 2; 1st night 4; 2nd day 6; 2nd night 6; 3rd day 11; 3rd night 10; 4th day 14; 4th night 9; 5th day 5 mm.

Light also exerts an influence; in the dark the stems become on an average 1—2 cm. longer than by daylight, which is chiefly attributable to a prolongation of the growth; light, then, shortens the grand period.[2])

The average daily increments of 6 specimens were in the dark: 2, 6, 10, 14, 15, 9, 3 mm.; in the light: 3, 8, 10, 15, 12, 2 mm., the circumstances being, for the rest, approximately the same; the 1st 59 mm. in 7, the 2nd 50 mm. in 6 days.

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[1]) H. FITTING, Jahrb. f. Wiss. Bot. 1919.
The decrease of the length is only slight in the plasmolytic solution; the correction for it may be neglected.

[2]) Cf. H. SIERP, Biol. Zentralblatt 1920, also J. H. v. BURKOM Thesis Utrecht 1913.

Phototropism (Heliotropism): As appears from observations in nature, the growing sporogonium-stems are very sensitive to phototropic and geotropic stimuli. Illuminated by unilateral clear daylight the long-stems curve distinctly already after ± 10 minutes, in the sunlight after 5 minutes.

The following observations were made in the dark room of the Botanical Laboratory of Utrecht [1]), where gas is absent; "laboratory air" caused no trouble; the objects, it is true, displayed many nutations, but this was quite the same in the open air. The temperature was 13°—15° C.; the observations were made with red light (electric lamp with safranin-bell). A preliminary test, in which the light was acting for hours at a distance of 1 or 2 d.m., proved that only a very slight curvature was brought about. This curvature was, moreover perhaps partly thermotropic.

The material, almost a pure culture, taken from the shady side of a ditch near Amersfoort, was cut into pieces with a sharp knife. These pieces just filled the ordinary boxes generally used for Avena. Subsequently these boxes were kept in the dark for a week at the very least, before they were used for the experiments. Only those objects which had stems of 15—25 mm. were used for the experiment, the larger ones were too feeble. Many of the sporogonia had to be removed before the experiment began, because they were crooked or displayed nutations; only the vertical stems were used, 20 of them serving as objects in every experiment [2]).

The curvatures were observed macroscopically; the black sporogonium indicated the deviation distinctly. In order to preclude any influence of chance nutations on the result, the time after which 10 out of the 20 objects displayed the curvature was considered as the experimental reaction time [3]). Unless the contrary appears from the experiment itself, the material was placed on the klinostat after the excitation (rotation time ± 6 minutes).

The candle-power of the various electric light-sources was measured with WEBER's photometer; some oscillations may have been caused by a modification of the net-tension. These, however, did not affect the principle of our research.

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[1]) Owing to the kindness of Prof. WENT this research could be continued during the holidays, which removed the difficulty that the growth of all the stems is completed in a few weeks.

[2]) Material grown specially for the experiment, would have yielded better results no doubt. This is in course of preparation.

[3]) W. H. ARISZ (Onderzoekingen over fototropie. Diss. Utrecht 1914) considers the "experimental reaction time" to be the time required for a curvature just noticeable to the unaided eye.

M. C. S.	M. C.	S.	Reaction	Exp. reaction time in minutes.
85	85	1	0	
170	85	2	0	
340	340	1	? 6 of 20+	
425	170	2½	? 9 of 20+	± 150
425	85	5	+ ? 11 of 20+	135
510	340	1½	+	130
510	85	6	+	135
612	0,01	612000	+	
850	85	10	+	130
1360	340	4	+	110
3400	340	10	+	105
7900	0,1	79000	+	
13600	1360	10	++	90
13600	3400	4	++	85
19600	11200	1¾	++	
27200	1360	20	++	80
33000	2200	15	+++	
34000	8500	4	+++	70
38250	2125	18	+++	70
40800	340	120	+++	
42000	700	60	++	
42000	2800	15	++ (*a.c.o.*)	70
56000	2800	20	++ (*a.c.o.*)	65
68000	340	200	++	
180000	50	3600	+	± 60
204000	3400	60	+	50
510000	8500	60	+ ?	
1020000	8500	120	+ ?	
1260000	700	1800	− ?	
2240000	11200	200	0	
3360000	2800	1200	at first + afterwards ?	25
7560000	6300	1200	„ + „ ?	20
± 13,5 million	11200	1200	„ + „ +	20
20 „	11200	1800	„ + „ +(*w*)	
31,5 „	17500	1800	„ + „ +(*a.c.o.*)	18
63 „	17500	3600	„ ++ „ +	15

The light from the source of 0.1 C.p. was dimmed by paper.

Control-experiments were made with a continuous illumination of 85 M.C., in which within an hour (55 or 60 minutes) a positive reaction was achieved. Then we ascertained the effect of several quantities of light-energy. The results are shown in the annexed table; they have been arranged according to the rising quantities of energy. The experimental reaction-time is expressed in minutes, the reaction is indicated by the symbols: 0 = no reaction; ? = doubtful reaction, curvature in fewer than 10 out of the 20; +? = faint positive reaction; + = distinct reaction; ++ = strong reaction, +++ = very strong reaction; −? = doubtful negative reaction.

In the experiments designated with (a.c.o.) the rays were transmitted through a dish filled with ammoniacal cupric oxide, (the walls of the dish running parallel) in the experiments designated with (w) the rays went through a dish filled with water.

As appears from the table the experimental threshold-value [1]) lies in the neighbourhood of 400 M.C.S.; this value might be determined with greater accuracy if the material were more homogeneous, and in that case it would perhaps prove to be lower. At any rate the sensitivity is here much smaller than in Avena.

The table also shows us the "product rule" for the threshold value [2]), further that the quantity of energy is invariably the decisive factor. Likewise it substantiates Arisz's conclusion that to every quantity of energy belongs a maximal curvature of definite magnitude and shape.

The large decrease of the experimental reaction-time with increasing energy is very striking here; for the threshold-value it is ± 150 minutes; for the highest quantities of energy it is only 15 min. (± 15 million M.C.S.). With long stems and unilateral clear daylight the time is ± 10 min., in the sunlight even less, viz. 5 min. The decrease of the exp. reaction-time is pretty regular, but in the neighbourhood of the weak reactions the duration is difficult of determination, or in Pellia this time might be broadly considered as an index of the strength of the stimulus. Whereas in Avena the shortest experimental reaction-time appears already between 50 and 100 M.C.S., it does not manifest itself here under 15 million M.C.S. Pronounced negative reactions were not noticeable in Pellia. We did observe, however, a retrogression of an appearing positive reaction (not past the 0-position, however), viz. at 3—7 million M.C.S. Between the strong positive reactions at 40000 and 15 million

[1]) Arisz l.c. and Rec. d. trav. bot. neerl. Vol. XIII 1914—1915.

[2]) Blaauw A. H., Die Perzeption des Lichtes Rec. d. trav. bot. neerl. 1909.

M.C.S. there also lies a field of no, or doubtful negative reaction at 1—2 million M.C.S. Lower down we shall endeavour to get a somewhat better insight into these facts, but for this it is necessary to study the photogrowth reaction, which after BLAAUW's research [1]) constitutes the basis upon which an interpretation of the phototropic phenomena has to be built up. Then the curvature is to be conceived as the resultant of the effects which the unilateral illumination has on the growth of the anterior and the posterior side. Beforehand I will communicate some other results of the phototropic examination.

First of all that with an illumination of 30000—45000 M.C.S., at which the reaction is strongest, almost hoop-shaped curvatures appear after 3 or 4 hours whether on the klinostat or not. Similar curvatures were observed by BLAAUW [2]) in Phycomyces, still more similar ones by ARISZ in Avena (l.c. Pl. I fig. 4). It would seem that in that case all the parts of the stem, which, as appeared above are all growing, although in various degrees, partake in the curving, evoked by the phototropic stimulus. After 16 hrs these curvatures have fairly disappeared also in the klinostat. This is the consequence of the autotropism, which VAN DE SANDE BAKHUYZEN [3]) looks upon as the consequence of the obscuration, which renders the process of growth-retardation, caused by the illumination reversible.

When a part of the stem is darkened by reed covered with tinfoil, the curvature is much less marked and occurs only in the lighted part. It proved impossible to illuminate the sporogonia alone; the tip of the stem was always lighted along with the other parts and displayed curvatures. When the sporogonium was darkened and the stem alone was lighted, a curvature appeared all the same. It follows then that when the sporogonium has been amputated, the stem does not react or reacts slightly phototropically, this is a consequence of the traumatic stimulus; there is no question here about conduction of the stimulus, which makes matters much simpler than in the case of Avena.

Here also the phototropically active part of the spectrum are the blue rays; when a glass dish filled with an ammoniac cupric oxide solution was placed as a screen before the light source, the effect was not altered appreciably, the experimental reaction time was about the same as when the dish is filled with water [4]).

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[1]) BLAAUW A. H., Licht und Wachstum I. Zeitschr. f. Botanik 1914.

[2]) Die Perzeption des Lichtes p. 345.

[3]) H. L. V. D. SANDE BAKHUYZEN, Analyse der fototropische stemmingsverschijnselen. Diss. Utrecht 1920.

[4]) The minimal influence of the red rays has been shown above.

The photogrowth-reaction: The space of time in which experiments with Pellia could be made, being very limited, I could make only few observations on photogrowth-reaction. They were performed with an energy of light of 75 M.C., the light falling from above through 4 mirrors arranged at an angle of inclination of $\pm 45°$. It was directed horizontally at 4 sides of the object, which was placed exactly in the centre of the 4 pencils of rays. In order to watch the growing object with the horizontal microscope, we could illuminate the field of vision by a fifth mirror arranged in the axis of the microscope behind the object, also at an angle of inclination of 45°.

The observations were always made with red light, so that no curvature could be effected when the various sides were differently illuminated. The black upper rim of the sporogonium was clearly distinguishable; unfortunately nutations most often caused difficulties again, because then crooked growth made the measurements unreliable. It appeared to be impossible to continue our observations longer than an hour.

The illumination lasted 5 or 10 minutes (Energy 22500 and 45000 M.C.S.). We chose this energy because it produced with unilateral illumination a strong positive curvature.

I will at once mention that with this quantity of energy Pellia yields a distinct photogrowth-reaction, provided the objects have been standing in the dark for some time. This also accounts for the phenomenon "disposition" (German "Stimmung"), that with unilateral illumination of 45000 M.C.S. objects, which have not been previously placed in the dark, or only for a short interval, do not yield a distinct curvature or yield only a very slight one, whereas the more sensitive objects (vide supra) react very strongly on it. Arisz l.c. was the first to conceive this phenomenon of "disposition" as a reaction-process and afterwards v. d. Sande Bakhuyzen assumed the "disposition" to be a difference in the slope of the growth-retardation curves, manifesting itself in every process in which the reaction does not increase rectilinearly with the stimulus. When the object was lighted omnilaterally with 2800 M.C.S. for six minutes (on the klinostat), and was subsequently exposed for 20 seconds to a unilateral after-illumination of the same energy, no reaction ensued, while a strong reaction does ensue, when a unilateral illumination with 56000 M.C.S. is applied, in which there is an equal difference of energy between the anterior-, and the posterior side.

Reverting to the photogrowth-reaction we observe after the addition of 4×22500 or 4×45000 M.C.S. in 5 or 10 minutes a distinct retardation of growth, appearing mostly during the illumination,

sometimes some minutes later. The mean retardation of 6 observations amounted to 35 % (min. 25 %, max. 47 %); the rate of growth seems to rise very slowly to the mean value before the illumination. Whether it rises beyond this value later on, was difficult to ascertain, since owing to the nutations the observations could be continued for an hour only.

1st object	12^h—$12^h\ 10^m$	dark	growth 85 μ	2nd object dark	70 μ
	$12^h\ 10^m$—$12^h\ 20^m$	,,	,, 87 μ	,,	73 μ
	$12^h\ 20^m$—$12^h\ 25^m$	light	66 μ	light	63 μ
	$12^h\ 25^m$—$12^h\ 30^m$	,,		dark	
	$12^h\ 30^m$—$12^h\ 40^m$	dark	64 μ	,,	40 μ
	$12^h\ 40^m$—$12^h\ 50^m$	,,	74 μ	,,	63 μ
	$13^h\ 50^m$—1^h	,,	75 μ	,,	66 μ

With unilateral illumination the anterior side of the still growing green parts is illuminated more intensely than the posterior side, which is due to absorption into the tissue. This was made out by photographs taken with sensitive paper of the silhouette [1]). The stronger retardation of growth at the anterior side, to which the unilateral illumination has assigned more energy, will therefore engender a positive curvature, in other words: the case of Helianthus hypocotylidons [2]). Only in Pellia the sensitivity is not so great and as yet no rise in the rate of growth beyond the normal has been detected during the oscillations, which may be called forth by the illumination [3]). To consider the first growth-retardation caused by the illumination as not belonging to the photogrowth-reaction (as SIERP [4]) does in Avena) makes no sense here, where conduction of stimulus is out of the question and the curvature may appear much quicker.

The results of the phototropic observations: the initial slow rise of the positive reaction up to ± 45000 M.C.S., the succeeding slow fall down to 1 or 2 million M.C.S., with a long stage of indifference without distinct negative reactions, where anterior and posterior side react alike, all this points, in my opinion, to a very gradual course of the growth-retardation curves. Probably the latter will show a

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[1]) Absorption is insignificant in the fully developed clear stems, but here also the curvature is slight.

[2]) A. H. BLAAUW. Licht und Wachtstum II. Zeitschr. f. Botanik VII 1915.

[3]) That also in Pellia the photogrowth-reaction must be accelerated after the growth-retardation might be inferred from the autotropism, the straightening in the dark.

[4]) H. SIERP. Zeitschr. f. Botanik XIII 1921.

very slow rise, then proceed horizontally over some length, (may be with slight falls), succeeded by an ascending portion. For the present it is impossible to give an accurate description [1]), which requires a detailed examination, with omnilateral fore-illuminations, succeeded by unilateral ones. I purpose to examine this more closely later on. Perhaps then also negative reactions will become evident.

Geotropism: Only few experiments were made regarding geotroprism; in nature it is negative in the stems. When the excitation was prolonged with vertical position of the thallus (stem horizontal), the experimental reaction time was 80—90 min. So we see that the difference in the rate of photo- and geotropic reaction is not so great as in Avena.

The experimental threshold-value for geotropism was ± 10 min., the experimental reaction-time 150 min.

By experiments with the centrifuge I tried to ascertain whether in geotropism the experimental reaction-time was equally dependent on the quantity of energy. The object appeared however unsuitable for this purpose, the flaccid stems are bent too much out of shape. Also the dishes proved impractical for this object. In experiments, on an inclined plane of 45° I found however, a much longer reaction-time.

Finally I wish to add a few remarks about the mechanical side of the curvature-process and the photogrowth-reaction in objects like Pellia, in which there is no question about conduction of excitation, and its progress, therefore, can be simpler than in Avena. When we try to find an explanation for the growth and the curvature of multicellular plant-parts, two factors may be considered; viz. change of the turgor-pressure or of the cell-wall. At one time the first was believed to be the chief factor, at present the second [2]). The turgor-pressure is, indeed, a necessary condition for the growth, but the changes of the cell-walls must be looked upon as the principal factors. This is borne out by the fact that during the winter-rest the osmotic pressure of the stem-cells is high in Pellia and still no growth reveals itself. On the contrary it begins and reaches its culmination point on the apex of the great period, when the osmotic pressure is regularly decreasing. It thus appeared that in comparing the convex and the concave sides of stems, which had just been curved, there was no difference in osmotic pressure worth mentioning. Nor does a distinct curvature brought about by unilateral

[1]) The slope of the growth-retardation curves is sure to decrease slowly, since with fore illumination the threshold value becomes higher.

[2]) W. Pfeffer Physiology, II. Cap. XIII.

daylight go back through plasmolysis already half an hour after the commencement of the stimulation. It follows then that here a modification in the nature of the cell-wall is answerable for the more or less considerable growth, which fact is quite in keeping with the explanation of the curvature of unicellular organs.

BLAAUW l.c. demonstrated in Phycomyces that photogrowth-reaction reveals itself only when the growing topzone of about 3 or 4 mm., is lighted. From this also he concludes that the light does not act through change of turgor.

The question now arises whether that change of cell-wall is primary or secondary, in other words: is the sensitive system to be found in the cell-wall or in the protoplasm. The latter is the more likely supposition, but it should be borne in mind that an investigation of R. HANSTEEN CRANNER [1]) lately informed us that the cellwall of the living cells is much more complicate than had formerly been suspected, viz. a complex colloid system which also contains lipoids. The whole problem of the growth of the cellwall, formerly interpreted through opposition and intussusception, will now have to be looked upon from a colloidochemical point of view.

More evidence has been produced, however, for the assumption that the sensitive system lies in the protoplasm, perhaps in the stable boundary layer. DREYER and HANSSEN's [2]) research showed that proteins coagulate under the action of rays of light of short wavelength. This action is a reversible process, as has been seen also heretofore for growth-reaction.

Be this as it may, in either case it is the chemistry of colloids which has to deepen our knowledge of the photo-chemical phenomena governing the photogrowth-reaction.

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[1]) R. HANSTEEN CRANNER, Jahrb. f. Wiss. Botanik 1914.

[2]) G. DREYER and O. HANSSEN, Compt. Rend. T. 145, 1907.
Cf. also F. SCHANZ Ber. d. d. Bot. Ges. 1918.

Mathematics. — "*Involutorial Correspondences* (2,2) *of the First Class*". By Prof. JAN DE VRIES.

(Communicated at the meeting of April 30, 1921).

§ 1. An involutorial correspondence (2,2) of the *first class* is characterized by the property that an arbitrary straight line contains *one* pair of associated points P, P^*. If we associate to each other the straight lines joining a point P to the two homologous points P_1 and P_2, also the field of rays is arranged in an involutorial (2,2). At the same time there arises a null system, if we associate to P the straight lines PP_1 and PP_2; each straight line has in this case two null points, each point has two null rays.

If the point P describes the straight line r, its null rays envelop a curve (r) of the fourth class that has r as a double tangent. The six points V in which $(r)_4$ is cut by r, are evidently *branch points* of the (2,2). The *branch curve* (V) of the (2,2) is therefore a curve of the order *six*.

We shall now suppose that the locus of the coincidences $P \equiv P^*$ is a curve of the order n. If P describes the line r, the points P_1, P_2 associated to P, describe a curve ϱ, which has the n coincidences on r and the pair of associated points on r, P, P^*, in common with r.

Through this correspondence r is therefore transformed into a curve ϱ^{n+2} of the order $(n+2)$.

Let us now consider the curves $\varrho_1{}^{n+2}$ and $\varrho_2{}^{n+2}$ corresponding to the straight lines r_1 and r_2. Besides the two points associated to $S \equiv r_1 r_2$ they have the points P in common for which P_1 lies on r_1 and P_2 on r_2; the other common points are *singular*, i.e. each of them is associated to ∞^1 pairs P_1, P_2.

The curves $(r_1)_4$ and $(r_2)_4$ corresponding to the straight lines r_1 and r_2, have in the first place the two null rays of the point S in common. The line r_2 cuts $\varrho_1{}^{n+2}$ in $(n+2)$ points P_2, which are associated to as many points P_1 on r_1 and accordingly define $(n+2)$ common tangents. The other $(12-n)$ common tangents are evidently *singular straight lines*; each of them bears ∞^1 pairs of points P, P^*.

Let us also consider the locus of the pairs of points P, P^* which are collinear with a point O. Let O_1 and O_2 be the points conjugated to O through (2,2); the curve in consideration ω is touched at O

by OO_1 and OO_2; it is therefore a *nodal* curve ω^4. Through O there pass six of its tangents; according to a theorem found by BERTINI the six points of contact, coincidences of the (2,2), lie on a conic [1]). The bearers of the *coincidences* of the (2,2) envelop consequently a curve of the *sixth class*.

§ 2. We arrive in the following way at a (2,2) for which $n=2$. Let the conic a^2 and the pencil of conics (b^2) be given. To the point P we associate the points P_1 and P_2 in which the conic b^2 through P is cut by the polar line p of P relative to a^2. On a straight line r, (b^2) defines an involution; as a rule this has *one* pair of points in common with the involution on r of the pairs of points that are harmonically separated by a^2. This (2,2) belongs accordingly to the *first class*.

The points of a^2 are evidently the coincidences of this (2,2). The straight line r is transformed into a *nodal* ϱ^4, which has the pole R of r as double point. For when P moves along r, its polar line p revolves round R and bears the two points P_1, P_2 associated to P.

The base points B_k $(k=1, 2, 3, 4)$ of (b^2) are *singular points*. On the polar line b_k of B_k (b^2) defines ∞^1 pairs of points P_1, P_2 which are associated to B_k. If P gets into the intersection of b_k with r, one of the points associated to P coincides with B_k; hence ϱ^4 passes through the four points B_k.

The conic b^2 through R cuts r in two points R_1, R_2, which are associated to R; hence ϱ^4 has a double point in R.

The six tangents of ϱ^4 meeting in R bear double points $P_1 \equiv P_2$; from this it follows again that the *branch curve* is a $(V)^6$. It has double points in the base points of (b^2); for the involution of the pairs of points on b_k associated to B_k contains two double points for which B_k is a branch point.

With a b^2 $(V)^6$ has four points in common besides the double points B_k; they are the branch points of the correspondence (2,2) on b^2. The curve $(V)^6$ touches a^2 in the six coincidences of the involution I^4 in which (b^2) cuts a^2.

§ 3. Any point A of a^2 is a coincidence of the (2,2), but it is also associated to the point A' which the tangent a at A has further in

[1]) Relative to this conic ω^2 as an invariant curve, ω^4 is transformed into itself by a central quadratic involution (inversion) with centre O of which the other two fundamental points lie on the polar line of O relative to ω^2; this straight line contains the points of contact O_1, O_2 of O. (See J. DE VRIES, *La quartique nodale*, Archives Teyler, série II, tome IX, § 12).

common with the b^2 through A. Of the locus α of the points A' a b^2 contains four points besides the base points B; they are defined by the points of intersection of b^2 with a^2. On each of the two tangents a through B_k, A' coincides with B_k; hence α has double points in B_k.

Consequently the curve in question is an α^6. As it corresponds point for point to a^2 and is therefore rational, it must have six more double points. There are therefore six points A' each corresponding to two points A; the b^2 through such a point A' cuts a^2 in the two points A which it has in common with the polar line of A'.

The straight line b_k is transformed by (2,2) into a β^4 with *triple point* β_k. When P moves along b_k the polar line p continues to pass through β_k, so that always one of the points P_1, P_2 associated to P, coincides with β_k. If also the second point is to coincide with β_k, p must touch the b^2 through P at β_k. Now any straight line p through β_k touches one b^2; if we associate the points Q_1, Q_2 which this b^2 defines on b_k, to the pole P of p, there arises a correspondence (1,2) between P and Q. Hence Q coincides three times with P; but then the curve β_k into which b_k is transformed, has a threefold point in B_k and is therefore a *rational* β^4. [1])

§ 4. We shall now try to find the locus of the *double points* $P_1 = P_2$. It has in the first place *threefold points* in B_k. On each b^2 there lie besides the base points four more points of the curve in question, namely the double points of the (2,2) in which the points of b are arranged. Consequently it is a δ^8. As it corresponds point for point to the branch curve $(V)^6$ it is just as the latter of the genus six; hence it must have *three* more *double points*. These we find in the double points of the three pairs of lines belonging to b^2.

The bearers of the double point $P_1 \equiv P_2$ envelop a curve of the sixth class (§ 1) of the same genus as the branch curve, hence with four double tangents; these we find in the straight lines b_k.

For the points where b_k is touched by two of the conics b^2, correspond as double points to the branch point B_k.

[1]) On b_k there lie 2 points that are associated in the (2,2) to each other and at the same time to B_k, and which therefore together with that point form a polar triangle of a^2. The b^2 containing them is consequently circumscribed to ∞^1 polar triangles so that on it the (2,2) has been transformed into a *cubic involution*. In this involution *each* base point B is associated to the points of intersection of b^2 with the polar line of B.

If we define the pencil (b^2) by two conics, each circumscribed to a polar triangle of a^2, each b^2 bears a cubic involution and the whole correspondence (2,2) is transformed into a system of ∞^1 involutorial triplets.

§ 5. Each straight line $B_k B_l$ is evidently *singular*, for it bears ∞^1 pairs of points that are harmonically separated by a^2.

A straight line would also be singular if the involution in which it is cut by (b^2), coincided with the involution of the pairs of points that are harmonically separated by a^2. And this will be the case when this straight line is touched in its two points of intersection with a^2 by conics b^2.

Now the straight lines t that touch b^2 at its points of intersection with a^2, envelop a curve of the class *six*. For the points of contact of the tangents out of any point to the conics b^2 lie on a cubic and this meets a^2 in six points, each of which defines a straight line t. This envelope is *rational*; it has therefore *ten double tangents*; to them belong evidently the six straight lines $B_k B_l$.

Hence there are, besides these, *four more singular straight lines*, s_k.

The straight line s_k is transformed through (2,2) into the system of s_k and a nodal cubic that has its double point in the pole of s_k. The straight line $B_k B_l$ is transformed into the system of $B_k B_l$, $B_m B_n$, b_k and b_l.

§ 6. The points P_1 and P_2 associated to P in the (2,2), correspond to each other in another (2,2), which may be called the derivative of the former. This $(2,2)^*$ is likewise of the *first class*; for on a straight line p there lies only the pair in which p cuts the conic b^2 passing through the pole P of p.

Also this $(2,2)^*$ has singular points in B_k; for if P describes the polar line b_k, P_1 remains in B_1 and P_2 describes the above mentioned rational curve β_k^4.

The curves ϱ_1^4 and ϱ_2^4 corresponding in the (2,2) to the straight lines r_1 and r_2, have (§ 1) 10 points P in common for which P_1 lies on r_1, P_2 on r_2. Hence P_2 describes a curve ϱ^{10} when P_1 describes the straight line r_1. This ϱ^{10} has *quadruple points* in B_k, for r_1 cuts the curve β_k^4 in four points P_1.

Each branch point of the (2,2) is at the same time a branch point of the $(2,2)^*$; accordingly they have also *the same branch curve* $(V)^6$. The *coincidences* of the $(2,2)^*$ are the *double points* of the (2,2); the *curve of coincidence* is therefore the above mentioned δ^8, which passes three times through B_k, twice through the double points of the pairs of lines. We find the points of intersection of r with ϱ^{10} in the eight points which r has in common with δ^8 and in the pair of points P_1, P_2 on r.

The four singular straight lines (§ 1) of the $(2,2)^*$ are found in the straight lines b_k.

§ 7. In the following way we arrive at a (2,2) for which $n = 3$. Let a^3 be a cubic, p^2 the polar conic, p the polar straight line of P. To P we associate the two points of intersection P_1 and P_2 of p^2 with p. The correspondence (2,2) arising in this way, is involutorial, because P and P_1 may be considered as threefold elements in a cubic involution where the points of intersection of PP_1 with a^3 form a group [1]), or as the double points of the cyclic projectivity defined by this group. The *class* of this (2,2) is therefore *one*.

If P gets on a^3, P_1 and P_2 coincide with P; P is in this case a branch point coinciding with the corresponding double point. If on the other hand P gets into a *point of inflexion* B, p is a part of p^2, so that B is a *singular point* and the stationary tangent is a *singular straight line.*

If P gets on the *Hessian* H^3 of a^3, p passes through the double point of p^2, also lying on the Hessian, and P_1 coincides with P_2, so that P is a branch point. The *branch curve* $(V)^6$ consists therefore of a^3 and H^3 and these curves are at the same time the locus of the *double points.*

When P describes the straight line r, p^2 describes a pencil and p envelops a conic. In each base point of (p^2) there lie therefore two points associated to P. As a p^2 contains moreover the two points of intersection with the corresponding p, the straight line r is transformed into a *quadrinodal curve* ϱ^5. This contains the nine points of inflexion of a^3, as these correspond to the points in which r cuts the stationary tangents. Consequently ϱ^5 touches a^3 in the three points of intersection of a^3 with r.

The derivative of this (2,2) is of the *fourth class.* For a straight line p has four poles and contains therefore the four pairs P_1, P_2 in which it is cut by the corresponding four polar conics p^2.

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[1]) KOHN, *Zur Theorie der harmonischen Mittelpunkte.* (Sitz. ber. der Akad. der Wiss. Wien, Bd. LXXXVIII, S. 424).

Botany. — *Light- and dark-adaptation of a plant cell.* By Dr. D. TOLLENAAR and Prof. A. H. BLAAUW.

(Communicated at the meeting of April 30, 1921).

Now that it has appeared that the growth in length of vegetative organs as a rule shows a very characteristic response to the light-stimulus, we possess in this response of growth to light an excellent criterion for the elementary study of sensitiveness to light. In this research of which all experiments have been made by Mr. TOLLENAAR, we have carried on the study about the way in which the light sensitiveness of an individual cell, the sporangiophore of *Phycomyces nitens*, reveals itself in the response of growth. The progress of the reactions is already known in case that fixed quantities of light of $^1/_4$ M.C.S. to 2 mill. M.C.S. are applied in a short time to cells that are kept in the dark (see Licht u. Wachstum I). Likewise how the response of growth is, when the cell already adapted to the dark is exposed to permanent light, e.g. of 1,64 or 4000 M.C. (see Licht u. Wachstum III). In 64 M.C. for instance we see acceleration- and retardation of growth interchange and gradually brought into equilibrium. When after this adaptation to light the growth has become constant again, its progress appears in 64 M.C. some percentages quicker than in the case of the cell adapted to the dark.

I. *Conversely, does the growth become some percentages slighter, when the cell after adaptation to light has been completely re-adapted to the dark in* $1\frac{1}{2}$ *or* 2 *hours?*

All experiments in this research have been made at 16° C. with horizontal and 4-sided illumination. For the cultures + stems were used.

Experiments of control. These are required, because the growth of the cells may somewhat change its speed in the course of 2 hours, even though the illumination remains constant, in consequence of chance causes or e.g. because the growth is still increasing or already decreasing. With cells that have been adapted to 64 M.C. the rate of growth differed after two hours, respect.: -1, $+3\frac{1}{2}$, $+1$, -1, $-7\frac{1}{2}$, $+7\frac{1}{2}$, $-2\frac{1}{2}$, $-1\frac{1}{2}$, -7, -5, $+3$, 0, $+1$, $+1\frac{1}{2}$, $+4\%$. average $-0{,}27\%$, i.o.w. after adaptation to 64 M.C. (in 2 hours) the average growth remained the same in the next 2 hours in 64 M.C

Experiments. 4 cells adapted to 64 M.C. and then left in the dark show after adaptation to the dark (after $1\frac{1}{2}$ — 2 hours) a change in growth of — 6.0 % (± 0.53): 14 cells from another series of experiments — 7,5 % (± 1,2). With one of these 18 cells the growth had increased after a 2 hours' stay in the dark, with two the growth was the same, with 15 decreased.

The cells from the dark, adapted after a few hours to light of

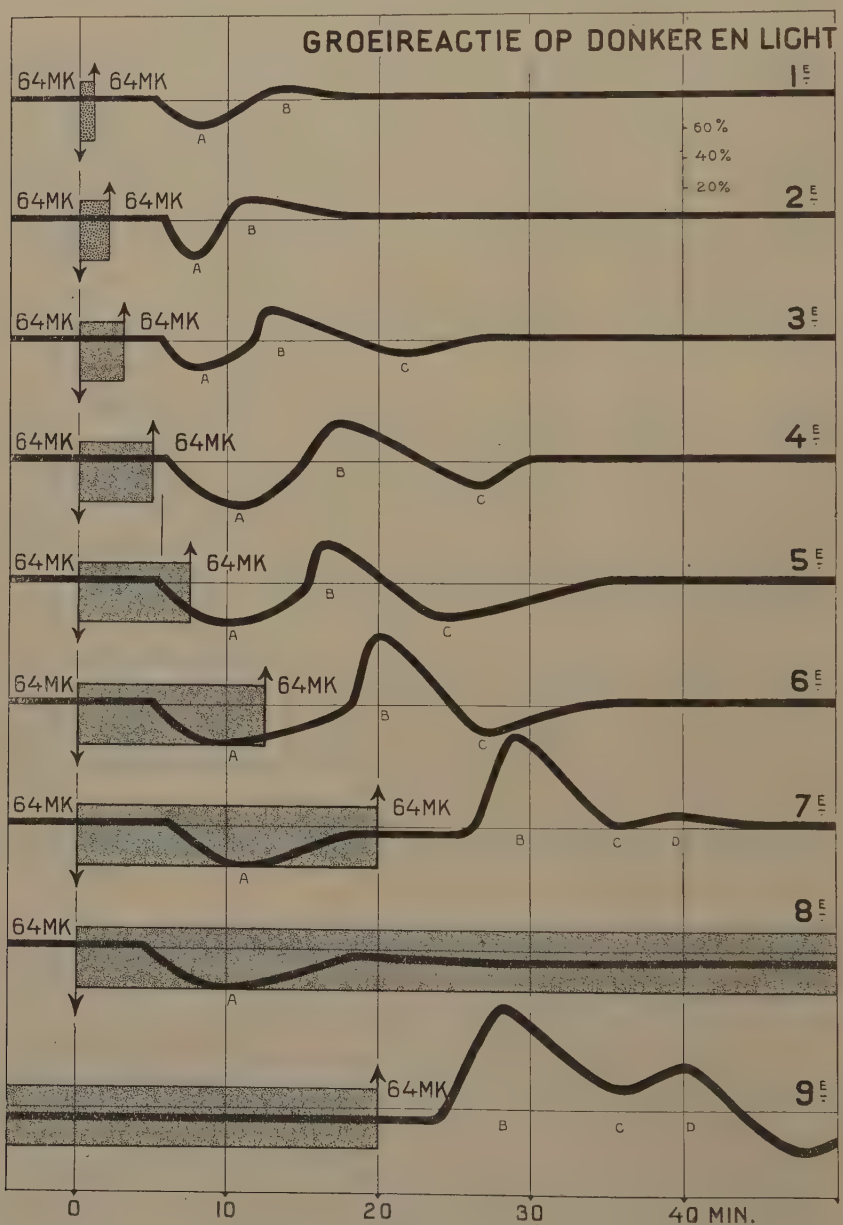

Fig. 1. Response of growth to dark and light.

64 M.C., grow some percentages quicker than in the dark; when left again in the dark, they constantly grow 4 — 10 % more slowly after dark-adaptation.

II. *Does the dark call forth a response of growth in a cell adapted to light, i.o.w. does the dark work as a stimulus, or is this decrease of growth of* 4—10% *in the dark quite gradual?*

Experiments. When the cell adapted to 64 M.C. is made permanently dark a decrease of growth sets in after $3^1/_2$—$5^1/_2$ min., so that after 8—$11^1/_2$ min. the rate has decreased to ± 73%; next there follows an increase, by which after $15^1/_2$—$18^1/_2$ min. the rate is about recovered ($98^1/_2$ %) in the light to decrease again a little and become gradually constant at ± 93% of the rate of growth in 64 M.C. (See fig. 1, curve 8).

So a disturbance of equilibrium takes place in consequence of the stoppage of the energy-supply, so that a typical response to stimulus ensues, which is contrary to the response of growth to light. If after the darkening the growth had *gradually* decreased to its dark-value, we could hardly have spoken of a response to stimulus. Now that there always occurs a reaction-time of $3^1/_2$—$5^1/_2$ min., just as with the response of growth to light and in consequence of the dark-fall an evident *disturbance of equilibrium* takes place, showing itself in *fluctuation of growth*, we may talk here of a typical *response to stimulus.*

For a cell adapted to constant light the dark (*sudden stoppage of light-supply*) *works as a stimulus. For some minutes the rate of growth in the light is maintained in the dark, then a sudden reaction follows, contrary to that which light causes.*

Responses to dark and to decrease of light have already been ascertained by Sierp for *Avena*. The response of growth to light of *Avena* is mainly a retardation of growth; the reactions observed by Sierp on dark- and light-decrease were accelerations of growth. It seems suitable to me to use the name of *dark-growth-response* for this phenomenon, as Sierp proposes with some reserve, provided an ample meaning is attached to the conception light- and dark-growth-response, viz. a response of growth to increase, resp. decrease of light.

Man states the result of the light-energy on the retina by himself much quicker through his impressions of brightness, than we can read the result of the light-stimulus on the metabolism in that cell in consequence of its change of growth. Moreover those processes

take place here at 16° C., with man at 37° C. Yet we are of opinion that we have every reason to see similarity between the way of reacting of the plant cell *to dark* and the appearance of a positive after-image, followed by the appearance of *negative after-images* in definite circumstances in the eye. There too after the fall of the dark a fluctuation of the impression of brightness, by which the impression of brightness or rather of darkness, with the negative after-image, may fall in the beginning *below* the normal darkness (or so-called intrinsic light of the retina) of the eye adapted to the dark. In this way the after-images are to be taken as a disturbance of equilibrium of the sight-apparatus adapted to light through the coming of the dark (= the stoppage of the energy-supply). Especially these negative after-images, which appear in our eye 2—4 minutes after strong prolonged illumination should be noted.

III. *What is the process of this dark-growth-response, when the cell has not been adapted to* 64 *M. C. but to slighter intensity?*

Experiments. Experiments were made in 8—1—$^1/_8$—$^1/_{64}$ and $^1/_{512}$ M.C. In 8 M.C. the average minimum of growth was 67 %. It further appears, that the growth also after slighter intensities of 1 and $^1/_8$ M.C. decreases to about the same value, viz. to ± 75 %. To be sure the reaction — just as with the common light-growth-response after weaker stimuli — appears *later*. The maximal decrease is from $8^1/_2$—11 min. after 64 M.C. and 8 M.C., shifted to 11—14 min. after $^1/_8$ M.C. After still slighter intensity of $^1/_{64}$ M.C., the growth decreases only to ± 85 %, after $^1/_{512}$ M.C. to ± 89 %. Therefore only after these slight intensities the dark-growth-response becomes clearly smaller, while the minimum remains just as after $^1/_8$ M.C. at 11—14 min. after the beginning of the darkening.

It is evident in these and other experiments as before, that the moments at which maxima and minima occur in the experiments with the various individuals are exceptionally constant. Especially with threshold-determinations when we can hardly say with certainty whether from a greater or smaller number an increase resp. decrease of growth may be inferred, the constancy of the points of time, at which the phenomena occur is a great aid in stating the appearance or non-appearance of an actual response. We want to point out in this connection, that Grünberg (1913) in his study on negative after-images was also struck with the uncommon constancy of the moments at which the after-images appear.

IV. *When the cell after adaptation to* 64 *M.C. is not permanently*

put in the dark, but the light is interrupted for a short time, what will be the process of the dark-growth-response?

Experiments. The light of 64 M.C. was in the various experiments interrupted by dark for 1, 2, 3, 5, 7½, 12½ and 20 minutes. A summary result is found in the following table, in which the principal moments have been given, while fig. 1 curve 1—7 shows the average process.

TABLE I. Response of growth of cells adapted to 64 M.C., in consequence of different times of darkness.

Darkening for	Beginning of response in min. after beginning of darkening	Minimum growth		Max. growth		2nd min.	
		after	in perc. of the growth in 64 M.C.	after	in perc.	after	in perc.
1 *min.*	4 —6½	7 — 9	85½ perc.	11½—14	103½ perc.	—	—
2 *min.*	5 —6½	7 — 8½	77½ „	10 —12	112 „	—	—
3 *min.*	4½—6½	7 — 9	83½ „	11½—13	118 „	19½—23	93 perc.
5 *min.*	5 —7	10 —12	74½ „	15½ 18½	124 „	25 —28	85 „
7½ *min.*	4½—6½	8 —10½	70 „	15 —17	126 „	22½—24½	79 „
12½ *min.*	4 —6½	8 —10	72 „	19 —21	145 „	26½—28	84 „
20 *min.*	5 —7½	10½—12½	74½ „	28 —29½	163 „	34½—37½	99 „
Perman.	3½—5½	8½—11	75 „	(15½ - 18½)	(98½ „)	± 94 perc. perman.	

In connection with fig. 1 the following may be observed, *Short times of darkness also call forth a typical dark-growth-response.*

With the short darkening of 1, 2, 3 minutes we were already struck with the fact, that after the minimum a constantly increasing maximum follows. (See esp. the table). When the cell remained in the dark, there was to be stated after the minimum also an increase of growth till just above the definite rate of growth in the dark, as had already been *previously* ascertained. But now that we darkened *for a short time* a maximum occurred, that becomes the greater according to the longer duration of the darkening, i.e. according as we put off the return of light.

It appeared *that a temporary darkening causes two successive reactions* resp. a dark-growth-response and a light-growth response.

In the successive experiments the *dark-growth-response* is mainly to be found on the same spot (see fig. 1 and table I) after 8—12 minutes. Only where the darkening after 1, 2, 3 minutes, i.e. very

speedily is superseded by light, the dark-growth-response cannot fully develop: the retardation of growth is earlier changed into increase of growth, so that the minimum is slighter ($77^1/_2$—$85^1/_2$ % instead of 70—75 %) and *consequently seems to be a little earlier* (after 7—9 min. instead of 8—12 min.). The beginning and the maximum of the light-growth-response on the contrary show themselves later with longer darkening, demonstrating in that way that it is the response to the return of light. The maximum was found with 1, 2, 3, 5, $7^1/_2$, $12^1/_2$, 20 minutes of darkening, resp. at $10^1/_2$—13, 8—10, $8^1/_2$—10, $10^1/_2$—$13^1/_2$, $7^1/_2$—$9^1/_2$, $6^1/_2$—$8^1/_2$, 8—$9^1/_2$ minutes after the return of light, while this maximum with complete adaptation to the dark (see Licht u. Wachstum III p. 102) has also been found at 7—$8^1/_2$ minutes after the beginning of 64 M.C. This latter reaction has been added for comparison as 9th curve to fig. 1. This shows that the first maximum, even the successive sinking and a second maximum observed at one time (1918) in permanent 64 M.C., now 1920 showed itself again after a darkening of only 20 minutes. The successive curves demonstrate, how by taking the dark periods longer and longer, we are able *to analyse the response to a short darkening in a dark-growth-response* (*A*) *result of dark-fall, and a light-growth-response* (*B*—*C*—*D*), *result of subsequent exposure to light.*

Meanwhile Sierp (1921) has considered with *Avena* the response of growth in short periods of dark. In this summary we can but refer to this. Though the transition from dark to light (64 M.C.) in the successive experiments is in a physical sense every time equally great, this transition causes an ever greater maximum in the growth (resp. $103^1/_2$—112—118—124—126—145 and 163 %), according as the cell has been darkened longer. By this it is already shown, that the cell adapted to 64 M.C. has greatly lost its sensitiveness and that the sensitiveness after the darkening increases very rapidly already from the first minutes. *This dark-adaptation* (= disappearance of light adaptation) is further shown by the following experiments.

V. *The adaptation to the dark of a cell used to light* may be demonstrated in two ways: *A.* by applying an equal quantity of light at different times after the darkening and considering how the response to this light-stimulus increases according as the cell has been longer in the dark; *B.* by determining how great the threshold of stimulation is at different points of time after the darkening.

Experiment A^1: A quantity of 256 M.C.S. (in 4 sec.) applied was

to cells, adapted to 64 M.C. at different times after the beginning of the darkening. See Table II and Fig. 2 curves 1, 2, 3, 4, 5. The first ± 15 min. after the darkening 256 M.C.S. has no effect on

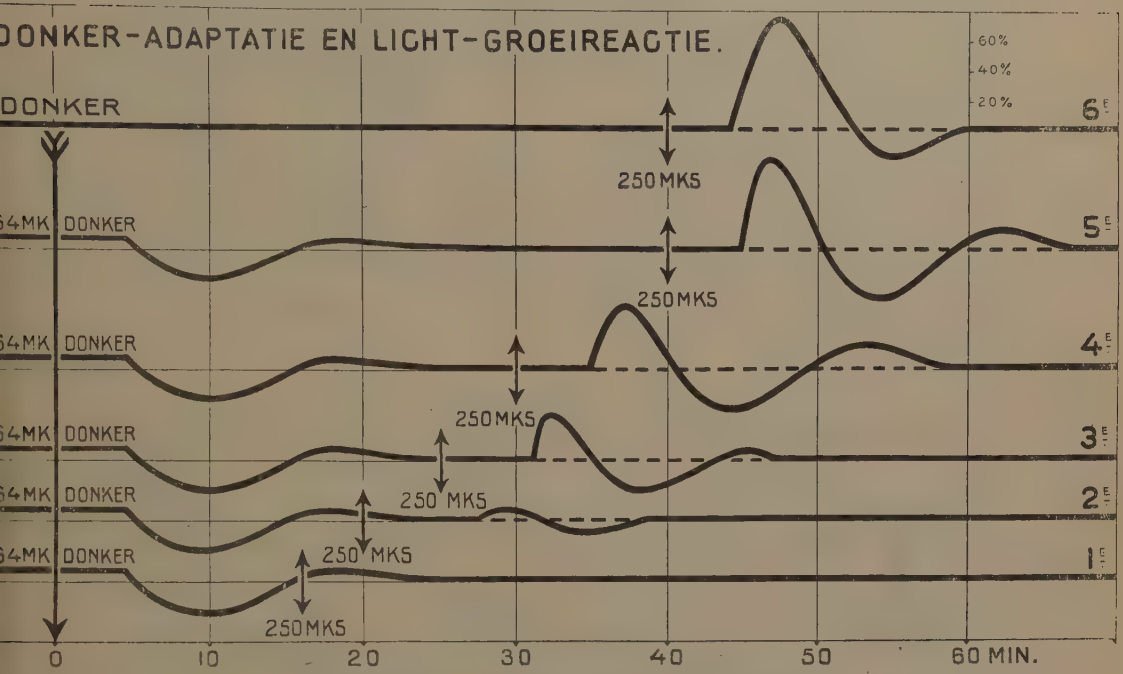

Fig. 2. Dark-adaptation and light-growth response.

the growth (curve 1), whereas this quantity does cause a maximal response by dark-adaptation (curve 6).

TABLE II. Response to 256 M.C.S. of cells, adapted to 64 M.C., at different points of time after darkening.

250 M.C.S. applied after:	Beg. of resp. in min. after exposure to 250 M.C.S.	Maximum		Minimum		2nd max.	
		after min.	in perc. of the growth in dark	after	in perc.	after	in perc.
20 min.	6½—8½	8 —10 m.	107½	13½— 15½	96	—	—
25 min.	5½—7	6 — 7½ „	132	12½—14½	83	19 —21½	105
30 min.	4 - 6	6½— 8 „	144	12½—15½	74	21½—24½	116
40 min.	4 —6	6 — 7½ „	160	12½—15	69½	20 —23½	112½
2 hours (full adaptation)	4 —5½	6½— 8½ „	176	14 —16	83½	—	—

Table II and fig. 2 clearly show the increasing adaptation in

consequence of the increase of response after longer darkening. Fig. 2 of course first shows the dark-growth-response, which has been left out in Table II.

Will the reaction begin earlier than by 256 *M.C.S. by exposure to* 1400 *M.C.S.?* This cannot be said with certainty, for cells adapted to the dark respond to 256 M.C.S. *for the eye* in growth more than to 1400 M.C.S. Moreover the question is, whether in the period of the dark-growth-response a light-response may be excited.

Experiments A'. In the first ± 5 min. after the darkening 1400 M.C.S. has no effect (*see curve* 1 *of fig.* 3), so that the response to the dark takes place as usual. But applied after 6½ min., light-growth-response already occurs, setting in therefore during the dark-growth-response.

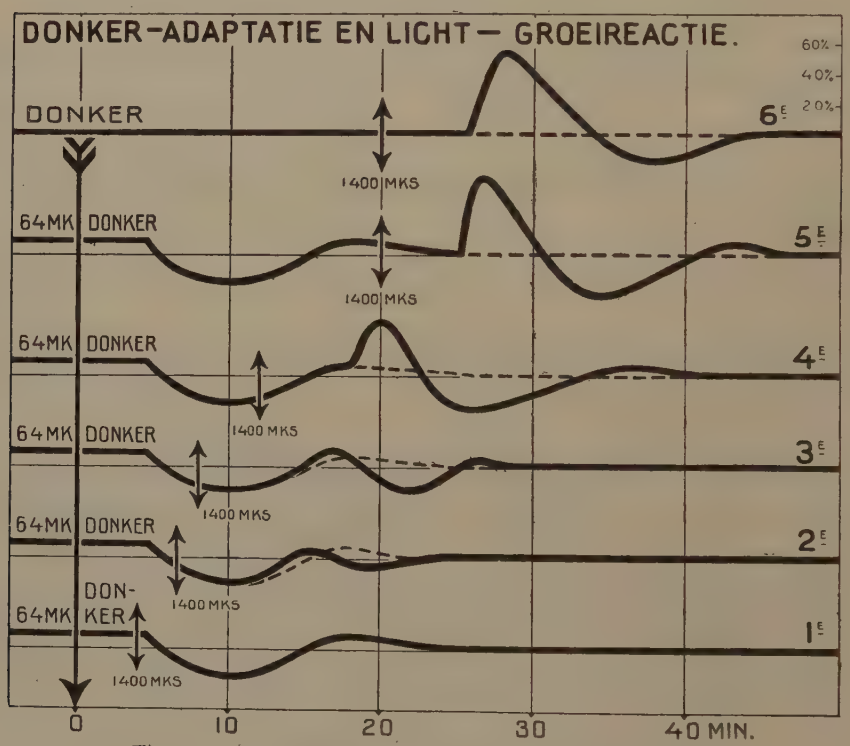

Fig. 3. Dark-adaptation and light-growth response.

See further fig. 3 and Tab. III. From this it appears in the same way as by application of 256 M.C.S., that the reaction grows stronger according as the cell has been further adapted to the dark. In fig. 2 and 3 the process of the growth is dotted, in case the latter light-stimulus had not been applied. Especially striking is the sudden sharp transition to the light-growth-response (see 3rd and 2nd curve).

After a darkening of 20 min. the response to 1400 M.C.S. has already much approached the response of cells adapted to the dark, which has been added for comparison in the 6th curve of fig. 3.

TABLE III. Response to 1400 M.C.S. of cells, adapted to 64 M.C. at different points of time after the darkening.

1400 M.C.S. applied after:	Beg. of resp. in min. after exposure to 1400 M.C.S.	maximum		minimum	
		after	in perc. of the growth in dark	after	in perc.
6½ min.	5½—7½	7½ 9½ min.	102	11½—13½ m.	98
8 „	6 —8	7 —10 „	110	13 —15 „	86
12 „	5 —7	6 —8 „	133	12 —14 „	78½
20 „	4½—6½	5½—7½ „	148	12½—15 „	73
2 hours (full adaptation)	5 —7	7¼—9¼ „	152	17 —19½ „	85

In the experiments of Tables II and III the process of the dark-adaptation — increase of sensitiveness — has *to some extent* been graphically represented by the percentages of the maxima of growth attained. Yet the quantitative proportion of the *sensitiveness* at different points of time of the adaptation-process has *not* been expressed in them, but the increase of the reaction-energy, being the result of that increased sensitiveness. But it is more important to express quantitatively the increase of the sensitiveness itself, and for that it is necessary to determine the quantities of light, causing an *equal effect* at different points of time of the adaptation-process, in order to make the sensitiveness inversely proportional. For this we prefer to choose the minimum effect, which is still perceptible to us, i.e. the limit or minimum-quantity, by which the light-growth-response occurs or with the classical term, the thresholds of stimulation.

Since it seems quite evident, that the effect of the light-energy in the cell with increasing stimuli gradually appears as response of growth and increases, we should be with this stimulus-process — and probably a great many others — careful with the tendency, lying in the word threshold. For convenience' sake we shall use the word here with that reserve.

Experiments B. In order to draw a comparison with the sensitiveness of the cell still completely adapted to 64 M.C., the threshold of stimulation was determined in cells being in 64 M.C.

When an additional 2000 M.C.S. was applied, no trace of a response

was found; when 3000 M.C.S. was administered (500 M.C. × 6 S.), in one of the 6 experiments a faint response was observed; with 4000 M.C.S. (1000 M.C. × 4 S) all 5 cells show a distinct response. *For* 64 *M.C. therefore the threshold lies between* 3000 *and* 4000 *M.C.S., i.e. at* ± 3500 *M.C.S.*

Next there were applied in the dark 2000 M.C.S. (500 MC. × 4 S), 256 M.C.S. (64 M.C. × 4 S), 32 M.C.S (8 M.C. × 4 S), 4 M.C.S. (1 M.C. × 4 S) $^1/_2$ M.C.S. ($^1/_8$ M.C. × 4 S), $^1/_{16}$ M.C.S. ($^1/_{80}$ M.C. × 5 S) and determined at what points of time these quantities are threshold-values. Moreover the threshold-value was determined for complete dark-adaptation. We had noticed that this was a good deal lower than the smallest quantity ($^1/_4$ M.C.S.) which was used before (see Licht u. Wachstum I).

The limit or threshold-value for the photo-growth-response of these cells adapted to the dark is at about $^1/_{100}$ M.C.S. This is a quantity much smaller than was hithertho used for stating vegetative reactions. By smaller quantities a reaction was sometimes perceived, but in the dark the limit is very difficult to fix, because with strongly decreasing quantity of stimulation the effect of growth decreases but slowly, about according to the cube-root of the quantity of stimulation (see L. u. W. I, which point we will further develop). So it already appears *that the cell in the dark is* ± 350.000 *times more sensitive for the light-stimulus than when adapted to* 64 *M.C.*

Table IV gives a survey of the process of adaptation from 64 M.C. to the dark.

TABLE IV. Process of adaptation or increase of sensitiveness after discontinuance of exposure to 64 M.C.

		Limit	Proportion of sensitiveness
In 64 M.C.		± 3500 M.C.S.	1
after 5 Min.	13 Min.	2000 „	1.75
„ 18 Min.	10½ Min.	256 „	13.6
„ 28½ Min.	12½ Min.	32 „	109
„ 41 Min.	14 Min.	4 „	875
„ 55 Min.	15 Min.	½ „	7000
„ 70 Min.		1/16 „	56.000
Adapted to the dark (after 90—120 min.)		± 1/100 „	350.000

In the first place it appeared, that also after the dark-growth response is finished and the growth after ± 30 min. has grown fairly constant, internally in the metabolism the dark-equilibrium has not been attained by far and will recover itself only after one and a half to two hours.

During the adaptation process the points of time may be defined fairly exactly for a fixed light-portion as threshold. For instance 32 M.C.S. gave after 25, after 26, after 27 min. no reactions, after 28½ min. three responded, one did not. In connection also with fig. 2 and 3 on the further increase of the reaction we see that the response of growth "turns up" quickly, when once the adaptation has sufficiently advanced for that light-portion. When however the dark-adaptation has for the greater part been attained, those time limits are fainter, since the sensitiveness does not increase so rapidly as during the full adaptation-process.

The rate at which the sensitiveness increases and rises to the end- or dark-sensitiveness, may be imagined by observing in the table that from 5 — 70 min., thus *during by far the greatest part of the adaptation-process, respect. in* 13, 10½, 12½, 14 *and* 15 *min. the sensitiveness becomes every time* 8 *times greater.* Between 18 and 28½ min. the geometrical rise appears to be strongest, between 0 and 5 min. it is geometrically yet a little slighter than between 5 and 18 min. By representing in a system of coördinates the times as abscissae, as ordinates the logarithms of the values of sensitive-

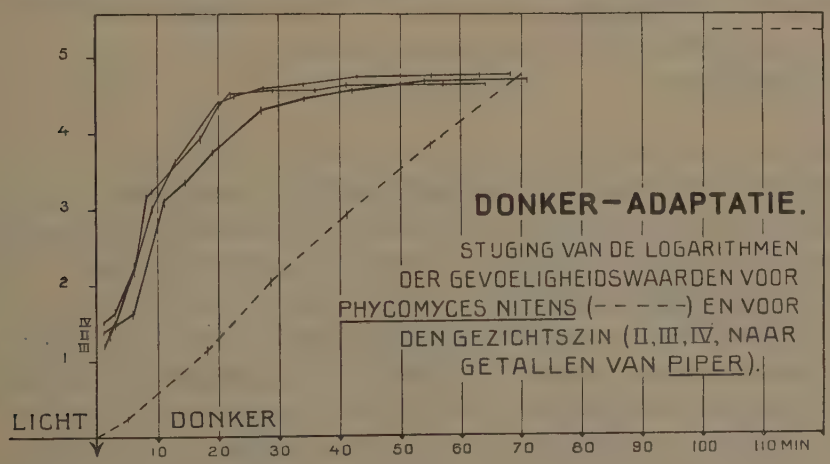

Fig. 4.

Dark-adaptation. Rise of the logarithms of the values of sensitiveness for *Phycomyces nitens* (. . . . .) and for the sense of sight (II, III, IV according to figures of *Piper*).

ness (see fig. 4), a good image may be got of the geometrical increase of the sensitiveness during the adaptation. In this figure the logarithm of the value of sensitiveness has also been represented, which is finally reached after complete dark-adaptation after $1\frac{1}{2}$ — 2 hours. When however as ordinates the sensitiveness itself is represented, the ascent of the curve shows the rate of the arithmetical increase of the sensitiveness as PIPER (1903) carries it out and discusses it for the adaptation of the sense of sight.

When therefore we would graphically represent the adaptation of these cells in the way of PIPER, it would give the impression (see Table IV), that there is but a very slight adaptation in the first 30 min., that after 70 min. only $^1/_7$ of the dark-adaptation is finished, and only after that the adaptation progresses fastest (curve steepest). If PIPER's adaptation-curves are converted, by representing the values of sensitiveness instead of the sensitivenesses themselves, and if they are compared with the same representation for the cell, the strongest rise of sensitiveness in man is found earlier than in PIPER's report and only then it becomes quite clear that the adaptation-process of our sight-impression is mathematically not so simple as with these cells. For three curves with an average course (II, III and IV of PIPER's observers) the logarithm of the values of sensitiveness has been represented in fig. 4. In the main 4 phases may be distinguished: a rather rapid one (first 3—6 min.), a very rapid one (3—6 to 8—12 min.) a rather rapid one (8—12 to 20—27 min.) and a very slow one (after 20—27 min.). When comparing we get the impression, that with the cell-adaptation we have to deal with a simpler process, though the same phases may be faintly distinguishable.

We have still to add that we determined the thresholds in these cells with fixed *quantities* of light, while for the human eye only *intensity*-thresholds have been determined. That makes the comparison more difficult. Determination of quantity-thresholds for the eye might picture the adaptation-process differently and more accurately. Moreover PIPER is wrong in not giving *the exact intensity* to which the eye was previously exposed, in beginning his first observations only after about 1 min. dark without an observation *in* light and in taking this first observation as *zero* in his curves.

Finally we may observe, that the width of adaptation with these cells is 1 : 350.000. With man it is according to PIPER only 1 : 2 to 8000; in consequence of the measuring of the intensity-threshold, and the mis-stated initial intensity a perfect comparison is not possible,

though it seems a great deal slighter than with the *Phycomyces*-cells.

VI. After it had already appeared in the research on the adaptation to dark, that the sensitiveness to light being in 64 M.C. is 350.000 times less than in the dark, we could further consider the course of adaptation when the cell has been previously adapted to fainter or stronger light. For the present however we had to restrict ourselves to the question: *How much changes the tone or degree of sensitiveness, when the cells have been adapted to different intensities of light*?

Experiments. After a stay of at least 2 hours in different intensities it was determined, what quantity of light was just able to call forth a light-growth-response, while the cells remain in that intensity. In Table V the result of these experiments is briefly summarized.

TABLE V. Proportion of sensitiveness after adaptation to different intensities.

Adapted to	Limit	Proportion of sensitiveness.
64 M.C.	3000—4000 M.C.S.	1
8 „	200— 400 „	8,75—17,5
1 „	25— 50 „	70—140
1/8 „	3— 6 „	580—1160
1/64 „	0,4— 0,8 „	4375—8750
1/512 „	0,1— 0,2 „	17.500—35.000
Dark	± 0,01 „	± 350.000

Now it appears, that for intensities of $^1/_{64}$ M.C. to 8 M.C. the sensitiveness decreases proportionally to the intensity to which the cell has been adapted. In 64 M.C. the sensitiveness seems to have lessened still more than would be expected according to this rule. To the very lowest intensities this rule could not hold good, because then the sensitiveness would become infinitely great in the dark. So we see after all that in $^1/_{512}$ M.C. the sensitiveness in comparison with $^1/_{64}$ has not increased 8 times, but only 4 times more. Yet it is already striking that it holds good to $^1/_{64}$ M.C.

One would be inclined to simply accept that one had to deal here with the law of WEBER. Yet we should be careful in making a comparison. We have here the very elementary case of one single cell, for which we have demonstrated as follows:

When the growth of Phycomyces-cells has been adapted to intensities of $^1/_{64}$ to 8 M.C., the quantity of light still calling forth a response of growth (the threshold of stimulation) rises proportionally to that intensity.

In verifying the law of WEBER two stimuli are compared with each other, which are unequal but applied in the same way, and the proportion is determined which is still observed as different. Here however the cell has been adapted to an intensity and we simply determine the *quantity* of stimulus which — applied in a short time — is threshold of stimulation with that adaptation. The cell therefore has been adapted to one stimulus, while the other stimulus is quickly applied as a portion. In fact this is another and more elementary experiment than the comparison of two intensities. As however the rule obtained so much resembles the law of WEBER for the comparison of two intensities, it is very probable that in point of principle we have to deal with the same phenomenon. Not in that sense that with the sensitiveness to light of this single cell there would be question of any psychical condition or power of discrimination, but conversely that these psycho-physical rules for the human perceptive faculty are at bottom based on simpler reactions in the individual cells to which those rules are already applicable.

It further deserves attention, also with a view to experiments and placing in so-called weak light, *that in a so slight intensity as* $^1/_{512}$ *meter-candle* these cells are already ± 15 times less sensitive than in the dark.

After these quantitative measurings of the adaptation or tone-change we have still to emphasize what follows. While the growth e.g. in 64 M.C. is only 6 or 7 % more than in the dark, so that for the rest such a cell at its growth cannot be distinguished at all from a cell in the dark, there is inwardly after adaptation to light quite a different condition, a different "*tone*". For that appears directly from the quantity of light that is wanted to induce such a cell adapted to light to a light-growth-response. The tone (condition of adaptation or degree of sensitiveness) is quite different and especially in this response of growth the phenomenon of adaptation appears in much purer form than in phototropical movements. The phenomena of tone already appearing in phototropical reactions and the subject of much study [see e.g. BLAAUW (1909), PRINGSHEIM (1909),

especially ARISZ (1915), BREMEKAMP (1918), v. d. SANDE BAKHUYZEN (1920),] occurring in consequence of longer exposures must not be exclusively taken as a consequence of the progress of the responses of growth.

It is perfectly true that part of the phototropical "tone"-phenomena may be solved as a result of the process of growth at the front- and backside (see v. d. SANDE BAKHUYZEN). But it should not be forgotten, that there is also a real change of tone which has a deeper base and is much more important in principle. In prolonged exposure real changes of tone (= phenomena of adaptation, i.e. changes in the degree of sensitiveness to light) *principally act a very important part* in part of *those* processes, which *underlie* the growth and *precede* the result of the growth. *The result* of this altered sensitiveness appears to us in the response of growth as an external symptom. We had better not use the word sensitiveness *of growth* (as we previously did occasionally, see L. u. W. III), because it might be easily forgotten, that at bottom sensitiveness to light rests with part of those deeper processes of metabolism, from which the growth secondarily results. Yet for convenience' sake we may call the growth (as well as the whole plant) "sensitive to light" as it is generally done in physiology with a number of phenomena of sensitiveness; provided we remember, that in most cases but some primary fraction of the larger complex is really sensitive to that factor and perceives it, while all the rest of the phenomena are only resulting reactions.

We have had to restrict ourselves to a summary of our research. The full data and a further discussion of the results and the literature will be published later on, while the researches on responses of growth are being continued.

Wageningen, March 1921.

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Experimental Psychology. — *"Psychical Inhibition"*. By Prof E. D. WIERSMA.

(Communicated at the meeting of June 25, 1921).

From daily experience we know that simultaneous sensations have an inhibitory influence on one another. During the day the light of the stars is not observed. Soft sounds, distinctly audible in the dead of night cannot be heard during the randiness of day time. The weak stimuli of the skin are nullified by the stronger. The psychical inhibition, under which is to be understood, that one psychical complex influences the other, so that intensification of the one entails a weakening of the other, was investigated by HEYMANS some twenty years ago. By numerous and careful experiments he determined the underlying principles. Although it is evident that the inhibitory process does not limit itself merely to sensations, it is important to determine experimentally that also other psychical complexes are subjected to similar laws. It is not easy to express this in definite measurements for all other complexes of consciousness, as ideas and emotions; but for volitional acts it is very well possible.

The investigation was carried out as follows: The individuals experimented upon were young, students and assistants. They were instructed to write down the numbers 1 to 25 as quickly as possible. The time was accurately registered by means of a $\frac{1}{5}$ second-metre. Repeatedly after approximately a quarter of an hour, when fatigue was out of the question, these experiments were repeated three times, once while a dynamometer was squeezed as tightly as possible, a second time while a foot was pressed against a fixed resistance, and thirdly while the teeth were firmly clenched. In all these experiments the dynamometer was held with the left hand to equalize the relations as much as possible.

The inhibitory influence from the movement of the left arm, the right leg and from the clenching of the teeth is obvious. The movement of the left arm inhibits most strongly, the clenching of the teeth less, and the movement of the right leg least of all. (See table).

In writing down successive numbers, an associative activity, which

was excluded in the following proofs, takes place beside the motion function.

Number of seconds with resistance in percentage of that without inhibition.

Subject.	Dynamom. Pressure	Foot Pressure.	Clenching the teeth.
H.	120.0	100.0	106.7
	128.6	107.1	114.3
	100.0	100.0	94.1
	118.8	100.0	100 0
D.	140.0	106.7	120.0
	135.8	100.0	100.0
	118.8	108.3	106.3
	131.3	106.3	106.3
G.	123.5	105.9	111.8
	111.1	94.4	105.6
	117.6	105.9	105.9
	123.5	105.9	111.8

The subject was requested to make as many strokes as possible. The number of strokes in 15 seconds was counted. These experiments were repeated twice, on the first occasion the dynamometer was squeezed continuously as tightly as possible, and the pressure was accurately registered on a kymograph; in the third instance the dynamometer was continuously pressed forcibly with the right foot, and the pressure was registered once more. To secure the same conditions the dynamometer was constantly held in the left hand, in each case including those in which it was not squeezed. In this way it is possible to ascertain, whether a disturbing influence affects the pressure of the hand, and the foot on the dynamometer in writing down strokes, and whether this pressure disturbs the latter. The disturbing influence of the pressure movement can reveal itself in the diminished number, but also in the irregular pressure. This can be registered by making use of the pen of Henri with which the pressure of each stroke act can be measured.

Experiments were performed on three individuals on nine successive days.

Number of strokes during disturbance in percentage of these in rest.

Percentage of strokes. Squeezing.	Percentage of strokes. Legpressure.	Percentage of strokes. Squeezing.	Percentage of strokes. Legpressure.	Percentage of strokes. Squeezing.	Percentage of strokes. Legpressure.
W.		H.		Ha.	
92.1	98.4	93.2	97.4	91.8	98.8
92.1	95.4	89.3	96.2	98.8	95.1
93.7	97.0	94.9	92.6	89.4	95.3
95.2	95.5	93.9	89.8	87.7	93.8
93.1	95.2	92.8	97.5	93.8	95.1
94.2	98.5	94.2	93.1	95.2	94.0
93.1	91.9	94.0	96.4	90.8	91.9
94.2	96.1	88.5	87.8	94.0	91.9
94.7	89.2	86.5	91.0	89.3	96.4

From the experiments it is apparent that no disturbing influence on the stroke test reveals itself in the pressure, which is practically always constant. On the other hand there is a marked disturbance in the number of strokes, and the left hand pressure constantly causes a more distinct disturbance than the leg pressure.

Moreover the stroke test causes a distinct inhibition on the squeezing of the dynamometer and on the foot pressure. The irregularity and the slow decline of the curve show this clearly. (See figures).

Fig. I. Foot pressure on dynamom. during stroke test.

Fig. II. Foot pressure on dynamom. apart from stroke test.

Further it was investigated to what extent a simultaneous, continous exertion of the left hand inhibits the squeezing effect of the right hand. It appears that the will-impulse from the left hand influences inhibitorily the squeezing effect of the right hand, when the former keeps in equilibrium a weight of 8 kg., which can be moved up and down over a pulley. The dynamometer was squeezed five times at intervals of 15 seconds each. At first the dynamometer was squeezed without disturbing influences, then with disturbing influences, and after a quarter of an hour the experiment was

repeated in reversed order. The percentages obtained under simultaneous will-impulses show clearly an inhibition during the three days on which the experiment was performed.

	Percentage of squeezing force during inhibition.
O.	86.2
	84.8
	89.7

After this result it was to be expected that the extent of inhibition would also reveal itself in the squeezing force. Therefore the same experiments were repeated on different individuals under an inhibiting pressure of 3 kg., 6 kg., and 8 kg.

Subjects.	Percentage during inhibition of		
	3 kg.	6 kg.	8 kg.
O.	95.4	91.4	82.8
	89.8	88.2	86.9
	96.9	95.4	89.3
Ha.	98.4	93.3	88.6
	97.7	91.7	82.5
	95.4	86.8	82.3
	93.2	90.1	85.3
K.	94.5	90.3	91.0
	91.0	89.0	87.5
D.	93.7	92.2	92.2
	100.0	96.2	96.2
G.	100.0	93.3	84.1
	100.0	102.7	94.6

From the above table it is evident that two simultaneous will-impulses exert an inhibitory influence upon each other, as was already determined for the sensations.

The question therefore arises, whether similar rules are applicable to subconscious psychical complexes. By numerous experiments it has been determined that the total contents of our consciousness embrace far more than we are aware of at a definite moment. Practice depends upon subconscious after-effects. It would be impossible to read and understand a book or to follow a lecture without such after-effects. Tact and experience are manifestations of subconscious psychical after-effects. Some experiments personally performed, and which exemplify the mutual influence of central consciousness upon subconsciousness, and of subconsciousness upon central consciousness I shall annex here. For these investigations hypnotic states are most suitable.

I instructed two individuals in hypnotic state to re-read six pairs of meaningless syllables, so often till they could repeat the paired syllable when only the first was mentioned according to the "treffer" method.

Subject.	Number of repetitions.		Percentage Gain.	Number of repetitions. Awake without previous hypnosis.
	Hypnosis.	Awake.		
C.	14	7	50	23
	15	9	40	21
	17	9	47	24
	22	10	54.1	20
	12	7	41.7	
	12	6	50	
	12	5	58.3	
	15	7	53.7	
F.	14	7	50	20
	17	8	52.9	23
	15	9	40	21
	14	9	35.7	22
	15	6	60	
	17	8	52.9	
	18	10	44.5	

Both subjects were in a state of somnabulism and could recollect

nothing of the experiment in a wakened state. An hour later I repeated the experiment in a wakened state with the same syllables. It appeared that the syllables could then be memorised with considerably less difficulty. This experiment was repeated several days every time with different pairs of syllables. To avoid the objection that one possibly memorises better in a wakened state, I made them learn by heart different syllables, in the same state during four days, at the same time of the day, and it appears that these must be read far more often in a wakened state.

The influence of subconscious psychical complexes is clear from the considerably diminished difficulty. Another clear proof of the influence of subconscious psychical complexes upon the consciousness is afforded by the following case. A patient in a rather advanced state of tuberculosis of the lungs complained of bilious colic which necessitated an operation. Some twenty years previously I had often treated the patient hypnotically, and therefore my colleague Koch consulted with me as to the possibility of performing the operation under hypnosis. We agreed to have everything prepared for narcosis in order to make immediate use thereof in case of insufficient effect of hypnotic suggestion. Fortunately this was not necessary. During an hour and a quarter the patient was kept under hypnosis, insensibility being suggested all the while. The operation complicated by synechia of the gallbladder was completed without the least disturbance. The patient did not give the slightest sign of pain for a single moment. Some time after the operation the patient awoke and recollected nothing of the operation. Nevertheless all that had happened during the operation appeared to be present in his subconsciousness, for brought under hypnosis anew he very well recollected that Prof. Koch had said: "Keep the hands off". "Here I have a stone". "Here is another". And no more had been said during the operation.

From this the existence of the subconscious psychical complexes is evident, but their influence upon the consciousness appeared clearly from another experiment. On account of his tuberculosis his doctor had forbidden him to smoke, but he seemed so addided to it that he smoked all day. To rid him of this habit I suggested to him under hypnosis that he had lost all desire to smoke, and since then (± one year) the patient does not smoke any longer. He has lost all inclination. He is unaware of the imparted suggestion, which he only recollects in hypnotic state and not in a wakened state. From this the influence is therefore evident of the subconscious suggestion upon consciousness.

Between conscious and subconscious psychical complexes there

exists merely a comparative degree of difference. The fluctuations in the attention have taught that weak sensations lie at times above and at times below the threshold of consciousness. One can voluntarily raise the degree of consciousness by removing the inhibitory influences. Differences in degree also exist between the conscious psychical complexes. If we have our attention concentrated upon a definite word, while we read, then we see this word distinctly, while we hardly observe the other letters.

Similarly we may assume that differences in degree also exist between subconscious psychical complexes. Some impressions still lie immediately below the threshold of consciousness, while others have sunk deep into subconsciousness. As the principle differences between conscious and subconscious psychical complexes are slight we may assume that the same rules may be applied to them. The inhibitory effects which have been determined for sensations and acts of volition will then be applicable to the subconscious afferent and efferent impulses.

From experience we know that cough and sneeze reflexes are greatly under influence of consciousness.

In case of a severe cold coughing and sneezing are usually immediately arrested when the mind is preoccupied during a lecture. It is remarkable that one coughs and sneezes much less in states of diffuse consciousness as during sleep. In the former case the threshold of the afferent stimulus is raised by the competition of other synchronous complexes of consciousness, in the latter by division of the psychical energy amongst numerous ideas.

Similarly the annoying secretion of mucus caused by a catarrh of the nose, in which the afferent stimulus remains altogether subconscious disappears or is appreciably lessened during intense preoccupation, and during diminution of consciousness.

A newly born child makes sucking movements as soon as the lips, or the soft palate are touched. This reflex afterwards disappears altogether, when the tongue and lips are used for other purposes. The sucking reflex has however not disappeared, but is merely inhibited, for under circumstances under which the inhibition of the voluntary movements disappears, as in an advanced state of Dementia Senilis or Dementia Paralytica the sucking reflex reappears.

The palm reflex of small children which is elicited, when the palm of the hand is touched, disappears at a higher age, when voluntary movements are more and more developed. This palm reflex reappears, when the inhibitory action of these voluntary movements is diminished, as, for instance, in the case of the greatly

lowered consciousness of Dementia Senilis and Dementia Paralytica. This also occurs when a lesion of the pyramidal tract disturbs the transmission of voluntary movements.

Pathology teaches us the same with regard to lacrimal secretion. The melancholicus, who is occupied by his dismal thoughts, hardly sheds any tears.

The above cases show clearly that consciousness has a constant influence upon certain reflexes. Other experiments bring clearly to light an inhibitory influence of synchronous efferent impulses.

A very young child shows the reflex of BABINSKI when the sole of the foot is stimulated. Later, when the child begins to walk, the toes are voluntarily flexed at each step, in order to stand firmer on the ground. Here voluntary movements become afterwards a reflex movement and by each stimulus of the foot sole a bending is elicited. Here too, the original sign of BABINSKI is not annihilated, but simply superseded, for in all cases, in which the efferent impulses of the plantar reflex cannot be elicited, and in which therefore their inhibitory influence has disappeared, the sign of BABINSKI is observed.

This is the case when the Pyramidal tract is disturbed and also in the case of diminished consciousness as normally in deep sleep of children up to the age of 13 years, and pathologically, for instance, in epileptic coma.

Since the mutual inhibition of conscious complexes is also applicable to mutual inhibition of subconscious psychical phenomena, the question is, in my opinion, justifiable, whether similar circumstances are not active in the cases of inhibition thus far poorly explained.

It is known that inhibitory influences are exercised upon the knee jerks by the cerebrum. Are there therefore synchronous motor impulses which inhibit the knee jerks? I should say the answer must be confirmative. A flux of impressions transmitting information as to the position of the limbs continuously travels from the periphery to consciousness. Consequently the easiest and pleasantest position is constantly assumed. At first that movement will probably be voluntary, but in course of time it becomes an involuntary and a reflex movement. We could therefore speak of the position reflexes, which are constantly present, and which must have an inhibitory action on synchronously elicited knee jerks. If it is true, that the position-reflexes exercise a disturbing influence, then the knee jerks must be higher, when the position-reflexes are absent or diminished. This actually appears to be the case, for the knee jerks of young children, where the position-reflexes do not yet exist, are exaggerated.

Only during later life, when the position-reflexes develop, the knee jerks gradually diminish. Further, a diminution in the position-reflexes is accompanied by exaggeration of the patellar tendon reflexes. The position-reflexes can be diminished under two circumstances. Firstly, the degree of consciousness of the centripetal stimulus can become weaker, and, secondly, the centrifugal motor impulse can be disturbed in its transmission. In both cases the inhibitory influence on the knee jerks will be diminished. The former occurs during strong pre-occupation, so that for instance the knee jerks are exaggerated in cases of hysteria, melancholia, or states of anxiety. Moreover, the centripetal stimuli will also be perceived less during a general relapse of consciousness as occurs normally during deep sleep or fatigue. Pathological relapses as Neurasthenia and states of dementia are usually accompanied similarly by exaggerated reflexes. The motor impulse of the position-reflexes will be disturbed in its transmission by lesions of the pyramidal systems, through which consequently the inhibitory influence upon the knee jerk is also absent.

In accordance with these facts, the knee jerks are diminished when strong impressions from without, emotions or other preoccupations, are as far as possible excluded. Then the adaptation to these perpetually centripetal impulses is greatest.

Other inhibitory influences upon the patellar tendon reflexes are also of great importance. SHERRINGTON has shown that each voluntary and each reflex muscle-contraction is accompanied by a synchronous relaxation of the antagonists. Here too, I should think the question must be raised, whether this relaxation of the antagonists is not to be regarded as an inhibitory influence from the simultaneous contraction of the agonists. A muscle is in a state of a certain tension, which is caused by a stimulus originating in the muscle itself. This muscle therefore is in a reflex tonus-state, which will diminish — analogous to the inhibition of simultaneous sensations or of simultaneous will-impulses, — when synchronous competing afferent stimuli arise, as these originating during the stimulus of the knee jerks, or when the simultaneous efferent impulses of the knee jerks are present. It appears to me, that the simultaneous relaxation of the antagonists during contraction of certain muscle-groups, which SHERRINGTON drew attention to, should be regarded in this manner. It is not even necessary that the agonists contract in order to acquire this relaxation of the antagonists. The arm of a patient with complete paralysis of the radial nerve was brought into such a position, that the slight tonus of the flexors maintained this position, and that without this tonus the arm would be extended by the force of

gravitation. If then the patient tried to extend the hand, this movement immediately took place. From this it is quite evident, that by the will-impulse to move the paralysed extensors the tonus of the flexors diminishes.

Would it not be possible that similar restraining phenomena accompany the act of micturition? In the bladder we have two muscles working antogonistically, the detrusor and the sphincter vesical. The empty bladder is shut off by a reflex closure of the sphincter. As soon as the quantity of urine, gathered in the bladder, is so large as to cause an efficient stimulus by distension of the bladder, the muscle fibres of the detrusor come into contraction. Simultaneously with this contraction of the detrusor, the sphincter relaxes. Here, therefore we have an inhibitory influence of simultaneous reflexes, just as for the patellar tendon reflexes. In this manner micturition sets in at regular times for newly born children. The reflex contraction of the detrusor can also be promoted by cold, and by other cutaneous stimuli, by emotions, etc. At a later age the peripheral stimulus is perceived as a desire, and as the child is brought up the attention is directed upon it. The desire can be voluntarily diminished or increased by distracting or concentrating the attention, and this is accompanied by the weaker or stronger reflex contraction of the detrusor and reflex relaxation of the sphincter. Micturition is therefore voluntary only in as far as the feeling of desire can be voluntarily increased or diminished to a certain extent by the concentration of the attention.

The influence of consciousness upon micturition is experimentally easily determined. During some days the quantities of urine were measured of some psychical normal and of some psychical abnormal individuals. The object of the experiment was kept cautiously secret. Only the quantities discharged simultaneously with defaecation, were not measured. (See tables following pages).

From these tables it is evident that during preoccupation and during diminution of consciousness, almost always greater quantities of urine are collected in the bladder. Due to distraction the desire is diminished, and consequently the contraction of the detrusor is retarded, and the restraining action of the sphincter is postponed.

When as a result of lesions of the spinal cord the desire is no more felt, and the micturition can no more be regulated, due to more or less concentration of the mind, then the bladder is again emptied regularly after certain filling. The quantities of each involuntary discharge of urine of a patient with total transverse lesion of the spinal cord is given below: (See Table)

NORMAL PERSONS.

Dr. H.	M.	Dr. W.	Miss P.	Miss F.	Miss J.	Mrs. S.
	500					
220	620	330	340	160	350	450
380	590	220	250	280	300	150
420	350	500	200	250	375	450
375	400	290	180	200	350	500
200	630	240	220	400	470	350
330	570	280	220	100	600	300
270	400	240	180	280	250	300
610	390	340	230	330	375	300
145	330	210	240	400	360	100
360	590	300	260	400	410	190
300	290	400	170	200	340	270
360	280	340	260	150	415	370
180	240	200	90	300	380	180
360	390	390	200	420	240	110
345	470	260	120	500	280	530

ABNORMAL PERSONS.

Melancholici.

Mrs. v. H.	Miss E.	Miss M.	Mrs. D.	Miss F.	Mrs. H.	Miss M.	de J.
1000	750	850	780	350	750	470	730
1450	500	450	1200	500	550	275	500
1200	450	700	400	500	500	300	540
1150	900	1000	500	750	550	230	325
700		600		700	510	600	450
1050				230	650	400	750
				800	725	820	320
					450	500	650
						670	

Melanch. C.	Melanch. Mrs. Pr.	Hysteria. Miss T.	Psychasth. Mrs. K.	Praecox, Stupor. Mrs. v. d. B.	Hysteria. Miss F.	Praecox, Stupor. Mrs. S.	Stupor. J. V.
	700	600	340	750	800	500	810
700	900	360	500	500	260	750	800
600	490	250	810	600	400	1650	820
1000	510	580	350	100	1000	600	770
750	1000	470	230	750	600	490	830
1150	400	730	550	750	270	750	960
1100	1100	340	350	450	800	740	630
1000	980	400	130	650		1000	570
990	1300	400	170	250		1300	490
910	1510	320	770	350		1550	620
	910	750	360	750		650	440
	890	400	90	650		500	
		80	610	600		1250	
		650		675		1100	
		560		700		750	
		600		700			
		700					

100	150	150	150	100	120
150	200	200	200	195	200
150	150	100	200	150	160
100	100	200	150	205	200
150	50	150	200	75	150
150	100	150	150	155	200
100	50	150	150	160	90
150	150	100	50	150	
100				160	

Must, therefore, all inhibition-processes be ascribed to synchronous competing influences? For this purpose a separate investigation into each phenomenon is necessary. I should like to touch upon one more, viz. the inhibitory influence upon the function of the heart through vagus stimuli. In 1845 the phenomenon was for the first time described by the WEBER brothers. Of this GLEY says: "Mais quelle est la nature intime de cette action? On dit que c'est un phénomène d'inhibition ou d'arrêt. Jusqu'à la découverte des frères WEBER l'idée d'excitation fut en physiologie étroitement liée à celle de mouvement; après cette découverte il fallut bien admettre que l'excitation d'un nerf centrifuge peut arrêter un mouvement. Alors la notion des phénomènes d'arrêt s'étendit peu à peu et elle établie aujourd'hui sur de nombreuses preuves. Mais nous ignorons toujours en quoi consiste exactement l'action inhibitoire". If this heart-inhibition is to be considered in the same manner as the above inhibition-processes, then there would have to be present two simultaneous motor impulses too. ROSENZWEIG, BOTTAZZI and GASKELL showed that in the auricle of the tortoise there is a muscle-layer directly beneath the endothelium, quite different from the rest of the cordiac muscle. By stimulation of the vagus the slow, rhythmical contraction of this muscle is intensified, while the rest of the cordiac muscle is weakened in its action. And, on the contrary stimulation of the sympathetic accelerates the heart action and inhibits this involuntary muscle-layer. Here therefore the inhibitory influence upon the heart by means of simultaneous efferent stimuli, exists in the same manner as in the case of the bladder function, the knee jerks etc. May we then assume the same for higher vertebrates? Here the circumstances are different, for we miss a so distinctly developed muscle-layer beneath the endothelium.

But it is however known that there exists another muscle-layer besides the cordiac muscle, viz., the bundle of HIS-TAWARA, which is found beneath the endothelium and ends in the fibres of Purkinje. In order to ascertain whether this bundle can contract the hearts of just previously slaughtered sheep were brought from the abattoir to the laboratory in their blood and warmly packed, on three occasions. In the laboratory a piece of cordiac muscle was immediately removed and placed in a chamber of ENGELMANN through which flowed defibrinated serum, previously oxygenated and kept at a temperature of 37° C. By means of an electrical stimulus a definite contraction in the cordiac muscle could be observed in this way. In two of the three cases it appeared histologically that a few cordiac muscle fibres were still present, but in the last experiment

these were wanting altogether. We may therefore assume that this tissue can contract. If we could ascertain that this bundle is innervated by the vagus nerve, we should have precisely similar circumstances as in the tortoise-heart. This however cannot be ascertained. The distribution of sympathetic and vagus nerve in the heart seems histologically uncertain.

I should think that for the mammalian heart we have reason to assume the possibility that here too the inhibition must be ascribed to simultaneous efferent impulses, in analogous manner as described for the knee jerks. Even if the vagus stimulus elicits but a slight contraction of the muscle bundle of His-Tawara, it is still very well possible, that this is the reason to which inhibiting influence upon the contraction of the heart is due.

Medicine. — "*Simple and alternating footclonus.*" By Prof. I. K. A. WERTHEIM SALOMONSON.

(Communicated at the meeting of May 28, 1921).

In ORDENSTEIN'S thesis, written under CHARCOT and published in 1863, the first exact description of footclonus is to be found. Since then it has been the subject of much investigation and of many publications. From these last it would seem that the mechanism as yet has not been fully elucidated. We know that footclonus is produced by rhythmical contractions of the calf-muscles, but there is some doubt about the behaviour of the anterior muscle group. Are these muscles at rest during the clonus, or is clonus caused by alternate contractions of the tibialis anticus group and the triceps surae? Is the clonus a simple or an alternate one? DUBOIS, working under CHARCOT, described it in 1868 as an alternate phenomenon. In 1875 it was represented as a simple clonus solely caused by the action of the triceps surae by ERB and by WESTFAHL. BLOCQ and ONANOFF are of opinion that the tibialis anticus actively participates in the movement. PETITCLERC says the same, but he thinks that the tibialis contractions cannot be felt(?). STERNBERG vindicates an alternating character only for the spontaneous footclonus, the ordinary footclonus being caused by rhythmical contractions of the soleus-group only. CROCQ is of the same opinion.

The observation of a patient in whom I was able to elicit a perfectly isolated clonus of the musc. extensor longus hallucis and likewise an isolated clonus of the m. tibialis anticus, caused me to construct an apparatus for graphically recording the contractions of the separate muscles of the leg. Once fixed to the leg, the apparatus could be used whilst the limb performed the most violent movements during a footclonus. A pair of metallic clamps were fastened on the leg, one resting on the upper part of the tibia, the other on the malleoli. A thinwalled brass tube connects the clamps. This tube proved to keep its position in relation to the tibia during any movements of the foot or of the leg itself. I could therefore use the tube as a support for two MAREY receiving capsulae, each bearing a small pelotte, which pressed on the muscle, the contractions of which were to be recorded. The thickening of the muscle was inscribed

in the ordinary way on a rotating smoked drum, at the same time as a time curve giving 0.1 of a second. The recording MAREY-tambours gave a magnification of the muscle thickening of only 3 times; the

Fig. 1.

recording levers weighed about 11 milligrams, their length being 7 centimeters.

With this apparatus (fig. 1) a great many records of footcloni were taken. As a matter of fact a visual and tactile examination was first made, and a graphical record was only taken either to make sure in a doubtful case, or more often to have a permanent record of some fact otherwise observed. In by far the most cases the apparatus was not needed, as during the clonus not the slightest contraction could be either seen or felt in the dorsal extensors of the foot or their tendons. Palpation in these cases is a very reliable mode of examination, especially if the hand be rested on the edge of the tibia. In the commencement of my series of experiments I made a good many records, just to make sure that a clonus was only a simple one. But I soon found that if no contractions could be felt, the recording lever invariably made a straight line.

In a few cases the tibialiscurve showed slight oscillations as in fig. 2. We easily find that these are not caused by real muscle

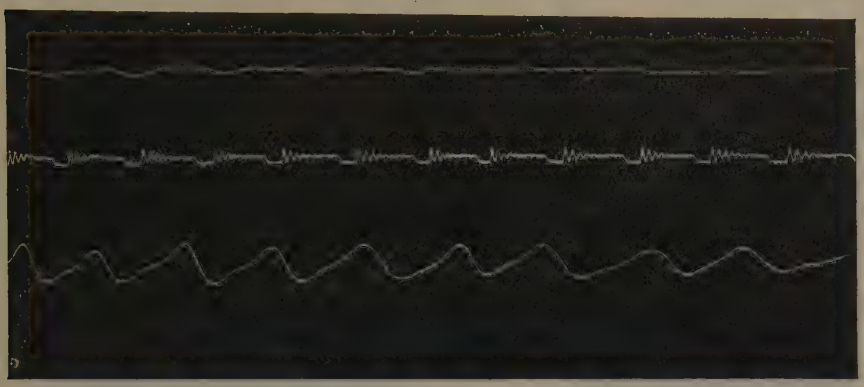

Fig. 2.

contractions, but by passive stretching of the muscle substance, the fascia and of the skin. The tibialis seems to thicken at exactly the same moment that the gastrocnemius contracts; this would mean isochronous contraction in the antagonistic muscles, which a priori would seem to be improbable. But the experiment shows that also a very slow passive flexion of the foot stretches the skin, the fascia and even the tibialis muscle itself, so as to cause a pelotte placed on the muscle to rise a little, at least if the pelotte be placed too near the knee. If it were placed too far away from the knee, much farther than the thickest part of the muscle, I obtained curves as in fig, 3, in which the passive extension of the tibialis shows itself by a slight downward movement of the recording lever. Between these two positions we are always able to find an area where the receiving tambour may be placed so as not to be disturbed by passive movements of the foot, whereas contraction of the tibialis anticus are faithfully recorded.

The placing of the receiving tambour on the gastrocnemius must also be done with the greatest care.

At first I placed it laterally upon the muscle; afterwards I preferred to apply it exactly in the middle line, slightly above the thickest part of the muscle. With the pelotte against the lower part of the muscle or even against the tendon it is impossible to get satisfactory records.

In contrast to numerous cases of simple footclonus we find every now and then a rare case in which inspection and palpation immediately show a participation of the anterior muscle group. Records of such cases of alternating clonus are shown in fig. 4—7. In all

these records the instrumental magnification is about the same, viz. about 3 times. The tracings show strong contractions, just as we feel them to be. We see from the curves that the contractions of

Fig. 3.

Fig. 4.

the calf-muscles alternate indeed with those of the tibialis anticus: the contractions of one group relax during the contractions of the other group.

The curves of the contraction of the anterior muscles as well as those of the gastrocnemius show a few peculiarities. The curve of

the gastrocnemius in fig. 4 — as in all the other figures the lower one — somewhat resembles a sinusoïdal line. The contraction- and relaxation-times are nearly equal, lasting about 0.08 second. At the highest point of the curve the m. tibialis abruptly begins to contract. The steep ascending part takes only 0.047 second, which is only about half the contraction time of the gastrocnemius. The tibialis contraction continues and remains constant for 0.096 second, after which relaxation sets in, occupying 0.076 second. The whole cycle takes 0.21 second, the clonus being an extremely slow one, of only 4.7 vibrations per second.

The difference in form and character of the gastrocnemius and tibialis curve is still more pronounced in fig. 5, in which the ascending period of the triceps curve lasts 0.077 second against 0.021 second only for the ascending part of the tibialis anticus curve. The frequency in this case is 7.3 per second. These figures found for the time occupied in the ascending and descending periods in the individual contractions do not agree very well with those generally found in simple muscle twitches. Yet the ascending part in tracings of simple twitches of the human tibialis anticus is always very small as compared with the relaxation time, whereas in curves of the triceps surae the difference is always much less noticeable. But may we compare a simple muscle twitch with a part of a clonus-curve? During a clonus the muscles are working under more or less abnormal conditions which do not resemble those under which records of "normal" muscle twitches are generally taken. During the footclonus we generally have to exercise a certain amount of pressure against the footsole. This means that, roughly considered, the gastrocnemius group are working against constant pressure, i.e. they contract isotonically.

The contractions might even be considered to be as nearly as possible isotonical ones, if the action of the anterior muscles could be excluded. This is more or less the case with the simple footclonus in which the soleusgroup for practical purposes may be said to work isotonically, as the disturbing influence of the mass of the patient's foot and the examiner's hand and arm will be hardly noticeable. The use of Ettore Levi's apparatus would bring us still a little nearer to perfect isotony. In cases of alternating footclonus we find that the passive tension on the triceps surae is rhythmically enhanced by the regularly occurring contractions of the tibialis group. Yet this influence of the tibialiscontractions on the tricepscurve is generally not very conspicuous, the gastrocnemius-group being many times stronger than the footextensor. Hence we may

expect that the inverse action of the contracting calf-muscles on the form of the muscle twitches of the tibialis anticus will be rather considerable. The tibialis anticus works against a tension which

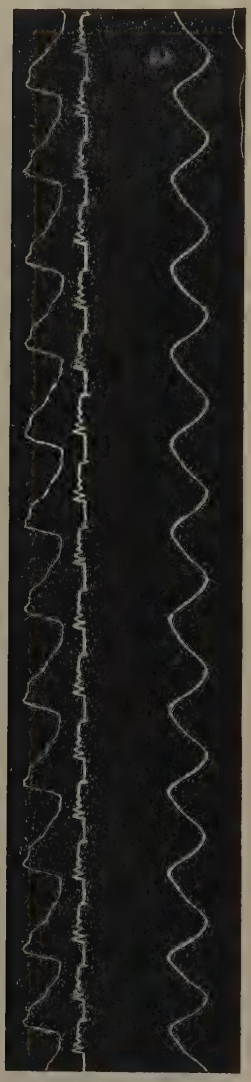

Fig. 5.

Fig. 6.

varies over a much larger amount, which may even become negative. The latter never happens with the triceps muscle. Consequently we expect the tibialis curve to be much more deformed than the gastrocnemius curve. On looking at our curves, we see that this is generally the case; but we also find that the influence of the two musclegroups on each other can easily be traced, though generally the action of

one group on the other dominates. In fig. 6 we see that the tibialis curve shows two successive tops. The first and larger one is the twitch of the tibialis proper, the second is caused by passive tension

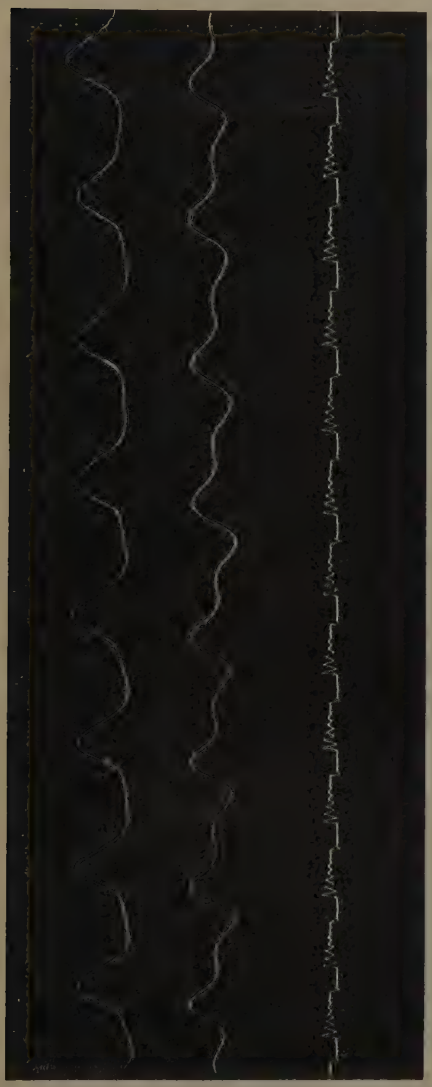

Fig. 7.

Fig. 8.

of the tibialis by the contracting triceps surae. In the triceps curve we see a slight elevation in the descending part of the curve caused by the triceps contraction.

In fig. 7 the tibialis curve is nearly undeformed as compared with an isotonic tibialis twitch; but the lower curve has in this case two clearly defined elevations, the higher of the two being the contraction proper, the lower one being caused by the tibialis contraction. How is the difference between the curves 6 and 7 to be explained? In both cases the apparatus had been correctly applied.

The difference was explained by considering the difference in force of the tibialis and the gastrocnemius contractions. In fig. 6 we have a clonus which is caused by the rhythmical contractions of the triceps surae and in which the tibialis anticus slightly participates. In fig. 7 it is the tibialis group that entertains the clonic movement which is but slightly assisted by the gastrocnemius. We conclude this from the way in which in both cases the clonus was elicited. In the case of fig. 6 the foot had to be forcibly pressed upwards. In the case of fig. 7 no pressure at all was necessary. I could start the clonus as well by a rapid passive extension of the foot as by a quick passive flexion. I may add here perhaps that in two patients I could elicit a clonus of the tibialis anticus alone, the triceps surae remaining completely at rest (fig. 8). With this record the pelotte for the gastrocnemius was placed on the middle and thickest part of gastrocnemius, in which I could not find the slightest trace of a contraction neither by inspection nor by palpation. The clonus could easily be started by a short passive plantar flexion of the foot. Both patients showing this clonus happened to suffer from incipient general paralysis. In one of them I also obtained an isolated clonus of the extensor hallucis longus.

The occurrence of a real alternating clonus as described in this paper is rather rare. I think that hardly more than 2 or 3 % of the cases of footclonus are such; we nearly always find a simple footclonus. My patients with alternating clonus suffered from multiple sclerosis, encephalomalacy or cerebrospinal syphilis; I also saw a few cases with apoplexia cerebri, braintumor, general paralysis, syringomyelia and atactic paraplegia. I do not think that I exaggerate in stating that every year I see about a dozen cases of alternating clonus against many hundreds of simple clonus. In complete medullary paraplegia I never saw alternating clonus. If we find it, it is always unilateral and invariably at the side of the lesser paralysis. The changes in the central nervous system were generally in both hemispheres and found to consist of multiple foci. No adequate explanation could be gathered from the localisation of the foci.

I have tried to explain the alternating clonus, starting from some facts about cerebral innervation stated by SHERRINGTON. Stimulation of a cortical motor centre causes a contraction of a definite set of muscles as well as a relaxation of the antagonists. The centre performs a rather complicated function. About a few of these centres we even know that their function is still more complicated, probably on account of the action of secondary centres: we find this with the movements of walking, standing etc. Probably on walking

a simple cortical impulse is given from a cortical centre to a lower one — probably in the thalamus — from which synchronous impulses are given to both extremities. For each step an impulse is necessary and in walking we get a succession of impulses alternately from the right and the left hemisphere, but each of them working on muscles of both legs. We may suppose that one centre innervates the contra-lateral extensors as well as the homolateral flexors. This means that a stimulation of the cortical centre causes a tetanic contraction of the contralateral extensors, hypotony of the contralateral flexors and hypertony of the homolateral flexors. Distruction of a centre of this kind is followed by hypertony of the contralateral extensors and also of the homolateral flexors. With a one-sided focus the muscles of the contralateral gastrocnemiusgroup are strongly hypertonic, the flexors slightly hypotonic but the muscles of the tibialisgroup of the homolateral side are also appreciably hypertonic. I think that this also causes the appearance of the shortening-reflex, described in a former paper. But at least we have also one condition favourable to the appearance of a tibialis clonus. As a rule it will not be possible to obtain a tibialis clonus as the large muscular mass of the leg and foot acts as a strongly damping factor, which immediately checks any commencement of a tibialis clonus. Only exceptionally, when a second focus in the other hemisphere has caused the appearance of a slight hypertonic state of the calf muscles, a commencement of tibialis clonus may be strengthened by a triceps clonus, or to an existing triceps clonus may be added a rhythmic clonic contraction of the tibialis group. If the second focus be an extensive one, the hypertony of the gastrocnemius group generally grows so as to entirely suppress the action of the anterior muscles. In all my cases the hypertony of the calf muscles was very slight. With a strongly hypertonic gastrocnemius I never succeeded in obtaining an alternating clonus. This sufficiently explains the rarity of the occurrence of alternating clonus.

Astronomy. — "*The local starsystem*". By Dr. A. PANNEKOEK. (Communicated by Prof. W. DE SITTER).

(Communicated at the meeting of May 28, 1921).

I.

If $A(m)$ denotes the number of stars of magnitude m, Δ the space density of the stars, which for a given line of sight is a function of the distance r, $\varphi(M)$ the luminosity-function, and if for the distance r we introduce $\varrho = 5\,log\,r$, thus making $M = m - \varrho$, then $\Delta(\varrho)$ can be found from $A(m)$, if both may be represented by quadratic-exponential functions. Thus if we put

$$log\,\Delta(\varrho) = h' + k'\varrho - l'\varrho^2; \quad log\,\varphi(M) = p + qM - rM^2;$$
$$log\,A(m) = a + bm - cm^2$$

we have:

$$h' = a - p + 3.786 - {}^1/_4\frac{(b-q)^2}{r-c} - \tfrac{1}{2}\,log\frac{r-c}{r}$$

$$k' = q - 0{,}6 + (b-q)\frac{r}{r-c}; \qquad l' = \frac{cr}{r-c}.$$

By these formulae (in a somewhat different form) KAPTEYN and VAN RHIJN have deduced the distribution of density in the starsystem surrounding our sun, representing it by a series of flattened surfaces of revolution. [1])

Here the function Δ has been found as a whole from the function A. But the observational data determine this function A for a certain extent of m only. Now the question arises, whether the value $A(m)$ for a given m determines the value $\Delta(\varrho)$ for a certain ϱ. The differential quotients

$$\frac{\partial}{\partial b}(h' + k'\varrho - l'\varrho^2) = -\,{}^1/_2\frac{b-q}{r-c} + \frac{r}{r-c}\varrho$$

$$\frac{\partial}{\partial c}(h' + k'\varrho - l'\varrho^2) = -\left\{\tfrac{1}{2}\frac{b-q}{r-c} - \frac{r}{r-c}\varrho\right\}^2 + \frac{log\,e}{2\,(r-c)}$$

show, that for $\varrho_0 = \dfrac{b-q}{2r}$ a variation of b causes not any, a variation of c causes only a slight variation of Δ; so that $\Delta(\varrho_0)$ depends nearly wholly on $a = A(0)$. If we count m and (in order to keep

[1]) J. C. KAPTEYN and P. J. VAN RHIJN. On the distribution of the stars in space...., Astrophysical Journal LII. 289.

the same $\varphi(M)$) also ϱ from the zero point m_0, this means, that in the formula

$$log\, A(m) = a + b(m-m_0) - c(m-m_0)^2$$

the first term $a = log\, A(m_0)$ determines $\Delta(\varrho_0)$ for

$$\varrho_0 = m_0 + \frac{b-q}{2r}.$$

Assuming that the observations determine the values for $A(m)$ of VAN RHIJN from $m = 4$ to $m = 16$ we find, by applying this formula, that they determine the Δ computed from them for $\varrho_0 = 10$ to 17 in the Milky Way (i.e. for $r = 100$ to 2500 parsec) and for $\varrho_0 = 9.5$ to 15 in the polar regions (i.e. for $r = 80$ to 1000 parsec).

As m_0 and ϱ_0 are conjugate values, it is only rational to take ϱ_0 as the zero point in the formula for Δ. If we put

$$log\, \Delta(\varrho) = h + k(\varrho-\varrho_0) - l(\varrho-\varrho_0)^2,$$

we have

$$\varrho_0 = m_0 + \frac{b-q}{2r}; \qquad \frac{1}{l} = \frac{1}{c} - \frac{1}{r}; \qquad k = b - 0{,}6;$$

$$h = a - 0{,}6 m_0 - p + 3{,}786 - \frac{(q-0{,}6)^2}{4r} + \frac{(b-0{,}6)^2}{4r} + \tfrac{1}{2} log \frac{r^2}{r-c}.$$

If we insert now the values $p = -2{,}394$, $q = +0{,}186$, $r = +0{,}0345$ [1]), we get:

$$\varrho_0 = m_0 + \frac{b-0{,}186}{0{,}069}; \qquad \frac{1}{l} = \frac{1}{c} - 29; \qquad k = b - 0{,}6;$$

$$h = a + 4{,}937 - 0{,}6\, m_0 + \frac{k^2}{0{,}138} + \tfrac{1}{2} log\,(l + 0{,}0345).$$

If over a limited extent of m the number of stars $A(m)$ may be represented by a quadratic-exponential formula it determines $\Delta(\varrho)$ over a limited extent of ϱ also. An adjacent extent of m affording a formula for A with other constants determines another part of the curve for $\Delta(\varrho)$. In case of an irregularly fluctuating course of $A(m)$ and $\Delta(\varrho)$ we may divide them into separate parts and represent each of them by such formulae, thus using the quadratic-exponential formula in an interpolatory manner. It may be noticed that in this case the coefficients c and l (which become zero together) may be negative, if only $l + r > 0$. Of course this solution of the problem to find Δ from A is not rigid, but only a practical and approximate one. If c approaches r very nearly, small errors in c cause enormous deviations in l, making Δ wholly uncertain; if c has a great negative value the result has no real meaning. If c surpasses the value $^1/_{29}$

[1]) KAPTEYN and VAN RHIJN, l.c. p. 297.

the solution becomes impossible; this points to discontinuities in the stardensities, (voids in the star masses, influence of distant starclouds) which we will not consider at this moment.

II.

By applying the above-mentioned formulae, in the following research we have tried to determine the shape of our local starsystem. The galactic zone between $\pm 20°$ latitude was divided into 12 sectors of nearly 30° longitude. The only sources giving sufficient data on the numbers of stars are the Durchmusterung Catalogues; we have used the counts of STRATONOFF [1]), giving the density per square degree for fields of 5° square according to the *Bonner Durchmusterung* (to 0° decl.), the *Südliche Durchmusterung* of SCHÖNFELD (to $-20°$ decl.) and the *Cape Photographic Durchmusterung*. As class 9.1—9.5 in all these catalogues is incomplete, only the numbers up to magnitude 6.5, 8.0 and 9.0 were used in our computations. The details of the extensive researches that were necessary to find the relation between these empirical scales and the photometric magnitudes will be given elsewhere; the resulting limiting magnitudes are for the different zones of declination of the N. hemisphere:

Decl.	0°—10°	10°—20°	20°—40°	40°—60°	60°—80°	80°—90°	
6.55 DM =	6.37	6.69	6.72	6.75	6.75	6.62	
8.05 „	7.97	8.15	8.26	8.29	8.23	8.10	all $-0.005\,(D-15)$
9.05 „	9.38	9.35	9.48	9.47	9.24	9.29	„ $-0.012\,(D-15)$

For the *Südliche Durchmusterung* we found

$6.48-0.014\,(D-15)$; $8.14-0{,}018\,(D-15)$; $9.39-0.025\,(D-15)$

where D denotes the number of stars per square degree up to 9.5 (not up to 10). To reduce these magnitudes to the scale, adopted in our computations, viz. the scale of *Groningen Public.* 18, corrected by the values, given by VAN RHYN in *Groningen Publications* 27 (G.P. 18 c.), we must still add to our results the values $-0{,}17$, $-0{,}08$, $+0{,}02$ for the three limiting magnitudes. For the C.P.D. for the galactic zone the values

5,76 7,86 and 9,46 (scale G.P. 18 c.),

were adopted; but these are much more uncertain than for the catalogues of Bonn.

From these $N(m)$, the number of stars from the brightest to the limiting magnitude m, the numbers $A(m)$, running nearly parallel

[1]) W. STRATONOFF. Etudes sur la structure de l'univers. Publications de Tachkent. Nr. 2 et 3. (1900, 1901).

with them, may be got by the relation $A(m) = \frac{d}{dm} N(m)$.

Putting

$N(m) = 10^{\alpha+\beta m+\gamma m^2}$ we get $A(m) = {}^1/_{\log e}(\beta + 2\gamma m)\, 10^{\alpha+\beta m+\gamma m^2}$, or

$$\log A(m) = \alpha + \beta m + \gamma m^2 - \log\log e + \log\beta + \log\left(1 + \frac{2\gamma}{\beta} m\right), \text{ thus}$$

$$a = \alpha - \log\log e + \log\beta; \qquad b = \beta + \frac{2\gamma}{\beta}\log e; \qquad c = -\gamma + \frac{2\gamma^2}{\beta^2}\log e.$$

For the mean magnitude m_0 was taken 8.0.

Further data are given by the *Selected Areas* of KAPTEYN; for each of the 6 Northern sectors the mean was taken of all selected areas lying in it. The numbers per half magnitude from 11,0 to 14,5 (for the greater part after the counts of VAN RHYN, kindly communicated to me), were doubled in order to represent the values $A(m)$ for $m = 11{,}25$ to $14{,}25$. They could be represented by linear formulae without perceptible curvature. In these formulae $\log A(m') = a' + b'(m' - 12{,}75)$ the m' denote photographic magnitudes; as for these faint classes the reduction of photographic to visual magnitude may be represented by $m - m' = -0{,}62 - 0.05(m' - 12{,}75)$, we have

$$\log A(m) = a + b(m - 12{,}13), \quad \text{where} \quad a = a', \quad b = {}^{20}/_{19}\, b'.$$

III.

The results of the Durchmusterung catalogues are collected in the next table, where the first column gives the mean galactic longitude of each sector and n the number of fields of STRATONOFF of 23 square degrees on the average.

long.	n	a	b	c	ϱ_0	h	k	l
15°	49	0.236	0.478	+0.0086	12.23	9.812	−0.122	+0.011
45	57	351	510	+ 0059	12.70	857	− 090	+ 007
75	48	312	480	+ 0171	12.26	971	− 120	+ 034
105	52	198	475	+ 0121	12.19	810	− 125	+ 018
135	56	147	446	+ 0212	11.77	932	− 154	+ 055
165	49	182	548	− 0126	13.29	540	− 052	− 009
190	38	246	556	− 0246	13.36	548	− 044	− 014
225	68	303	488	− 0017	12.38	793	− 112	− 002
255	48	264	481	+ 0030	12.27	793	− 119	+ 003
285	52	272	503	+ 0032	12.59	9.767	− 097	+ 004
315	68	277	440	+ 0271	11.68	0.204	− 160	+ 127
350	37	102	466	+ 0049	12.06	9.771	− 134	+ 006

For the sectors 315°, 135° and (in a lesser degree) 75° the great positive value of c, approaching r, giving a great value of l that raises also the value of h, causes a strong maximum for Δ, rapidly decreasing on both sides; so we find here a condensation of stars in space. The empirical data upon which it depends, consist in a strong curvature of the A-curve, i.e. a strong increase of the stars of the 8th magnitude, not continuing in the same manner for the 9th magnitude. As in this case small variations in c bring about great variations in Δ, it is necessary to examine the reality of these condensations as to the uncertainty of the starnumbers N, which have by chance distribution a mean error $\mu = \sqrt{N}$. Therefore for these sectors corrections corresponding to this uncertainty were applied in the direction of diminishing c and then the computation has been repeated [1]). Now the results are:

long.	*a*	*b*	*c*	ϱ_0	*h*	*k*	*l*
75°	0.304	0.489	+0.0095	12.39	9.869	−0.111	+0.013
135	0.138	0.455	+ 0134	11.90	9.803	−0.145	+ 022
315	0.266	0.446	+ 0219	11.77	0.062	−0.154	+ 060

The condensation at 135° has wholly disappeared; the great density at 75° joins the dense parts in sector 45°; the condensation at 315°, however, remains evident and does not disappear even by greater corrections to *log N*. Unless, perhaps, the scale of magnitudes strongly deviates here from the mean galactic zone, this condensation must be considered as real.

A deviation in opposite sense is shown by the sectors 165°, 190° and (in a smaller degree) 225°, where c and l are negative. Because the number of stars increases from the 8th to the 9th magnitude at an unusual rate — it is known that on this side of the sky the stars of magnitude 9—10 are strongly condensed over a fourth of the galaxy — we find that the space density at first, in the vicinity of the sun, decreases rapidly, but at a greater distance ceases to do so; whether it increases afterwards, cannot be decided by these data.

The results of the *Selected Areas* for the 6 Northern sectors are collected in the next table, where n denotes the number of the areas used (each being $^4/_9$ square degree).

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[1]) The coefficient c will be diminished, if $N(6,5)$ and $N(9,0)$ are increased, $N(8,0)$ is diminished. A small calculation showed that we get an even chance if for the value of this increase or diminution we take $0{,}6\,\mu$.

long.	*n*	*a*	*b*	ϱ_0	*h*	*k*
15°	7	2.045	0.360	14.65	9.391	—0.240
45	7	2.195	354	14.56	562	— 246
75	6	1.991	301	13.80	567	— 299
105	7	1.981	387	15.04	238	— 213
135	5	1.845	358	14.62	197	— 242
165	7	2.094	352	14.54	468	— 248

While the Durchmusterung Catalogues determine Δ on the average for ϱ from 11,5 to 13,5 ($r = 200-500$ parsecs) the *Selected Areas* determine it between 13,5 and 16 ($r = 500-1600$ parsecs). In the main the values for Δ from the *S.A.* fit close to the values from the *D.M.* An exception is made by sector 165°; here the Δ curves intersect for $\varrho = 14{,}4$, $log\ \Delta = 9{,}5$, but they show a different course: according to the *S.A.* the density is strongly, according to the *D.M.* it is hardly decreasing. In this sector the number of stars shows a discontinuity, as the strong increase for the magnitude 9—10 does not continue in the lower magnitudes of the *S.A.* Presumably this is caused by the influence of a remote galactic stream, which must be studied in another way.

IV.

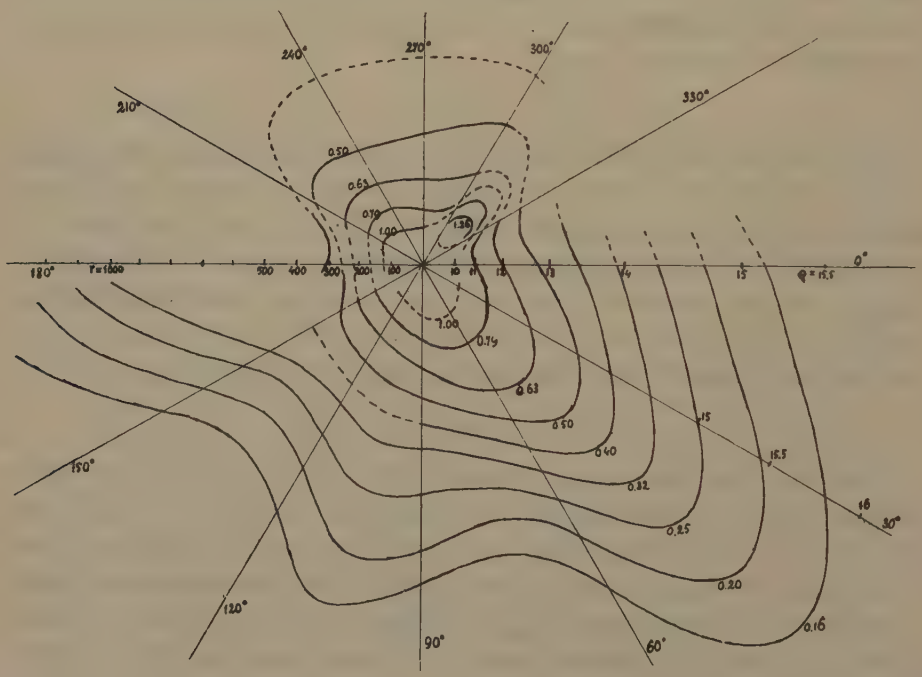
In order to find from these results the distribution of stardensity along the galactic section of the local system, values of ϱ were deduced from our formulae, for which $log\ \Delta$ gets the values 0,0 9,9 9,8 etc. In order to have no more irregularities in the resulting figure than are probably real, the results of the second computation were used for the sectors 75°, 135° and 315°. The values falling beyond the limits of the formula are placed in parentheses.

(*See table following page*).

These values have been used for the construction of the figure showing curves of equal density, decreasing with 0,1 for log Δ (at the curves the values of Δ itself are put down); where ϱ is not determined by the data, but has been extrapolated, the curves are dotted.

On the Southern hemisphere, where the density could not be found at greater distances than 500 parsecs, we perceive in sector 315° (Scorpio) the condensation, mentioned above, at a distance 100—200 parsecs, where the density surpasses 1,25. For the rest the central mass with density 1 extends farthest in the direction $l = 60°$; at greater distances the maximum density lies in the sector 45° (Cygnus), where a starstream seems to issue from the system.

long.	*log* Δ =	0.1	0.0	9.9	9.8	9.7	9.6	9.5	9.4	9.3	9.2	9.1
15°	ϱ =		(10.4)	11.5	12.4	13.1 / 13.4	13.8	14.2	14.6	15.0	15.4	15.8
45			(10.7)	12.2	13.4	14.0	14.4	14.8	15.2	15.6	16.0	(16.4)
75			(11.0)	12.1	13.0	13.4	13.7	14.0	14.3	14.6	15.0	(15.3)
105			(10.0)	11.4	12.3	12.9	(13.4)	13.8	14.3	14.7	15.2	15.7
135			(10.0)	11.2	12.0	12.6	(13.1) / (13.0)	13.4	13.8	14.2	14.6	15.0
165						(11.1)	12.2 / 14.0	14.4 / 14.4	14.8	15.2	15.6	(16.0)
190						(11.4)	12.4					
225		(9.8)	10.6	11.5	12.4	13.2	(14.1)					
255		(9.6)	10.4	11.4	12.2	13.0	(13.8)					
285			(10.0)	11.1	12.2	13.3	(14.3)					
315		11.4	12.1	(12.6)								
350			(10.2)	11.1	11.9	12.6	(13.3)					



Whether the remote fluctuations between 75°—150° are real, is not certain; perhaps the inward bending of the curves in sector 135° and 350° is caused by the absorbing nebulae in Taurus and Ophiuchus. Also in the direction 180° the starsystem extends to a great

distance; but here the discontinuity already spoken of, and the lack of counts for fainter stars makes the explanation of the data uncertain.

Of course the results of this first investigation have only a provisional character, and that for two reasons. In the first place by the incompleteness of the data: while up to the 9th magnitude the Durchmusterung Catalogues afford a rather complete though coarse material (by the uncertainty of the reduction to the photometric scale), we have hardly any data for the 10th and 11th magnitude. For the fainter classes the Selected Areas form an excellent but very limited material, while it is uncertain in what degree the local irregularities vary the average values for greater regions. Thus we do not know the whole course of $A(m)$ from the 6th to the 14th magnitude, which would be necessary to remove all doubts on the course of $\Delta(\varrho)$.

In the second place it must be emphasized that by taking together extended space-sectors with artificial boundaries the real irregularities in the distribution of the stars, with perhaps wholly different boundaries may be partly effaced, partly changed in their character. Moreover by regarding the influence of near absorbing nebulae and of remote galactic objets on the number of stars the results for space density may still be modified.

Physics. — *"Gravity and Pressure of Radiation."* By H. GROOT. (Communicated by Prof. W. H. JULIUS.)

(Communicated at the meeting of June 25, 1921).

§ 1. In 1910 LEBEDEW succeeded in experimentally showing pressure of radiation on *gases*, and measuring the value of the pressure. Since then attempts to take this force into account in astro-physical researches have not been wanting.

Particularly EDDINGTON [1]) and JEANS [2]) have reached highly remarkable results concerning the structure of "giant stars" by introducing besides gravity, also pressure of radiation into their equations. They come, among other things, to the conclusion that through the influence of this pressure, the gravity in the interior of a star can be considerably diminished, and this the more as the density is smaller.

One is naturally led to extend this investigation to states as will probably be met with in nebulae. And this the sooner as different authorities advocate the hypothesis that the law of NEWTON is not valid during the nebulous stage of a star, i.e. during the period that the star is being formed from primitive nebular matter.

KAPTEYN [3]) and CAMPBELL [4]) tried to account in this way for the surprising fact that the proper motions of the stars increase as a more advanced spectrum type is examined. The latter indicates a possible pressure of radiation as a force that might partially neutralize gravity.

Also F. NÖLKE [5]) in his cosmogonic considerations has recourse in numerous places to the pressure of radiation to render the not being constant of gravity plausible.

An estimation of the *extent of the possible effect* is, however, nowhere found. And so long as this is wanting all conclusions which are exclusively based on qualitative speculations, remain unreliable — as but too clearly comes to light in the different cosmogonies.

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[1]) M. N. **77**, (1916—17), p. 16 and p. 596; Astrophys. J. **48**, (1918).

[2]) M. N. **79**, (1919), p. 319.

[3]) J. C. KAPTEYN, Astrophys. J. 1910 (April).

[4]) CAMPBELL, Lick-Observ., Bulletin. Vol. VI N°. 196.

[5]) „Das Problem d. Entwicklung unseres Planetensystems", 2te Ausgabe (1919) Berlin; A. N. **188**, (4509).

What follows may be taken as an attempt to get some insight into the quantitative relations.

Here three different problems present themselves, which we will discuss successively:

A. To what pressure of radiation is a nebula subjected from the stars scattered around it (system nebula-star)?

B. What pressure does an absorbing body (planet) experience from the radiation of an extensive nebula in the neighbourhood?

C. Can the parts of a nebula repel each other appreciably through mutual radiation?

§ 2. *The system: nebula-star.*

When we wish to make an estimation of the relation between the attractive forces, to which a nebula is subjected from the surrounding stars, and the repulsive forces caused by the radiation of these same stars, we may begin by remarking that it is independent of the scattering in space of the stars considered. For the two forces are in the ratio r^{-2}, hence their ratio is not influenced by the distance. As not all the stars of the same absolute magnitude have the same mass, it would practically be necessary for the determination of the resultant of the active forces to know the nature of each of the stars concerned accurately. This is, of course, impossible. In our investigation we shall assume that on an average all the stars have an equally large mass, and radiate equally strongly as our sun. On this simplified supposition the ratio of the attraction of the whole stellar system to the repulsion caused by the radiation of the same system, is equal to that of the same forces exerted by one star at any distance.

With a view to the hypothesis of KAPTEYN and CAMPBELL mentioned before [1]) we will examine the following case more closely.

A star with a mass equal to that of the sun may be at 1 parsec. distance from a spherical [2]) nebulous mass of a radius of 15000 astronomical units. Seen from the star, the nebula occupies the 0.0014^{th} part of the sky.

Let us assume in order to find an *upper limit* of the pressure of radiation to which the nebula is subjected, that *all* the radiation received from the star, is absorbed. (We know that in reality the absorbed fraction is exceedingly small). The star emits as much

[1]) Compare also the view of H. SHAPLEY, Astrophys. J. 50, (373), 1919.

[2]) I choose the spherical form to simplify the calculations; one should not think here of a *planetary nebula*, which is known to show on the other hand very quick proper motion.

radiation as our sun, i.e. $4{,}2 \cdot 10^{33}$ ergs per second. The energy absorbed by the nebula per second is $5{,}1 \cdot 10^{29}$ ergs. The pressure of radiation can be calculated in this case with the well-known formula:

$$D = \frac{S}{c} \quad \ldots \ldots \ldots \quad (1)$$

in which S = quantity per second of received radiation in ergs, $c = 3 \cdot 10^{10} \frac{cm}{sec.}$, D = pressure of radiation in dynes.

This yields in our case:

$$D = 1{,}7 \cdot 10^{19} \textit{ dynes.}$$

Putting the mass of the nebula, like that of the star, about equal to that of the sun, i.e. $2 \cdot 10^{33}$ gr., we find for the maximum acceleration through pressure of radiation:

$$a = 0.8 \cdot 10^{-14} \frac{cm}{sec^2},$$

for that of the attraction:

$$a' = -1{,}4 \cdot 10^{-11} \frac{cm}{sec^2}.$$

Accordingly by the side of the attraction the pressure of radiation, even when calculated on exceedingly favourable suppositions, is almost negligible. As the same ratio must be valid with regard to the whole stellar system, we conclude:

The attraction of the stellar system on a nebula is not appreciably modified by pressure of radiation. Deviations from the law of Newton in such nebulae as we have considered, cannot be accounted for by the counteraction of the pressure of radiation.

Of course this consideration no longer holds when the dimensions of the nebulae become hundreds of times greater. But for the problem in question we were obliged to assume that the nebula from which the new star is being formed, had already conglomerated to the stated dimensions.

§ 3. *The system: nebula-planet.*

In view of some cosmogonic considerations on the origin of the solar system, it may be of interest to examine how great the pressure of radiation is which can be exerted on a newly formed planet by the mother nebula.

We begin by solving the question: what is the pressure of radiation which a spherical nebula of *constant* density ϱ and radius R

exerts on a planet P, which absorbs all the received radiation, and which is at a distance Δ from the centre of the nebula.

An element $d\tau$ of the nebula emits in the direction OP a quantity of radiation given by:

$$\frac{Se^{-\mu\rho s}\cos\psi}{x^2}\,d\tau$$

when $\mu =$ absorption coefficient, $x =$ distance from $d\tau$ to planet, $s =$ length of the path passed over by the radiation inside the nebula, $S =$ the intensity of radiation, which we shall assume to be constant inside the nebula (see figure).

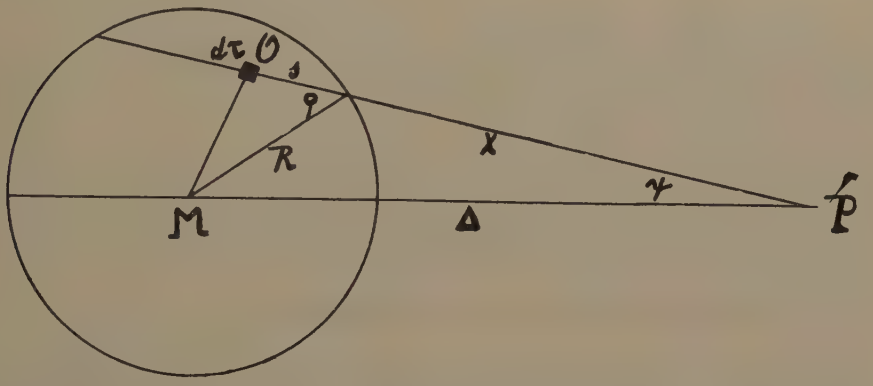


Fig. 1.

When $d\tau$ is taken $= x^2 dx \sin\psi\, d\psi\, d\omega$, and $r_1 =$ radius of the planet, the radiation of the whole nebula on the planet is:

$$A = \pi r_1^2 S \iiint dx\, d\psi\, d\omega \sin\psi\cos\psi\, e^{-\mu\rho s} \quad . \quad . \quad . \quad . \quad (2)$$

When we introduce φ instead of ψ (see figure) through:

$$\Delta \sin\psi = R\sin\varphi$$

and when we take $dx = ds$, (2) passes into:

$$A = \frac{\pi r_1^2 S R^2}{\Delta^2}\int_0^{\frac{1}{2}\pi} d\varphi \int_0^{2R\cos\varphi} ds \int_0^{2\pi} \sin\varphi\cos\varphi\, e^{-\mu\rho s}\, d\omega \quad . \quad . \quad . \quad (3)$$

Integration yields [1]):

$$A = \frac{\pi r_1^2 \cdot 2\pi R^2 S}{\Delta^2}\cdot\frac{1}{\mu\varrho}\left[\tfrac{1}{2} - P^{-2} + P^{-1}e^{-P} + P^{-2}e^{-P}\right] \quad . \quad (4)$$

$$P = 2\mu\varrho R$$

[1]) Compare: Bottlinger, "Die Gravitationstheorie und die Bewegung des Mondes", Bayerische Akademie, 1912.

When M = mass of the nebula, m = mass of the planet, then by making use of:

$$M = \tfrac{4}{3}\pi R^3 \varrho \quad , \quad m = \tfrac{4}{3}\pi r_1^3 \varrho',$$

we may write:

$$A = \frac{Mm}{\Delta^2} \cdot \frac{9S}{4\varrho\varrho' r_1} [\tfrac{1}{2} P^{-1} - P^{-3} + (P^{-2} + P^{-3}) e^{-P}] \quad . \quad . \quad (5)$$

The pressure of radiation experienced by the planet when it absorbs all the radiation received, is again calculated with the formula:

$$D = \frac{A}{c}.$$

The NEWTONIAN attraction must be diminished by this amount in order to find the resulting force K.

This becomes therefore:

$$K = f \cdot \frac{Mm}{\Delta^2}\left[1 - \frac{9S}{4fc\varrho\varrho' r_1}(\tfrac{1}{2} P^{-1} \quad P^{-3} + P^{-2}e^{-P} + P^{-3}e^{-P})\right]. \quad (6)$$

Can this diminution be great enough to bring about appreciable disturbances?

To investigate this, the following hypothetical case may be considered:

We assume that the solar nebula, after the formation of Neptune, has withdrawn to within the orbit of Uranus. We suppose Neptune itself to be still gaseous, though considerably denser than the solar nebula, and with a radius a 100-times larger than at present.

We further disregard the fact that the solar nebula in all probability must have had already a pretty great central condensation, a circumstance which has an unfavourable influence on a possible effect of pressure of radiation.

We base our calculation on the following numerical values:

Radius of Uranus orbit	$R = 2{,}868 \cdot 10^{14}$ (cm.).
Present solar radius	$R_\odot = 6{,}96 \quad 10^{10}$] (cm.).
„ solar density	$\varrho_\odot = 1{,}4.$
„ Neptune radius	$r_0 = 2{,}8 \quad 10^9$ (cm.).
„ density of Neptune	$\varrho_0' = 1{,}1.$
Gravitation constant	$f = 6{,}66 \cdot 10^{-8}.$
Absorption exponent[1])	$\mu = 0.0002.$

Thus we find:

$$K = f\frac{Mm}{\Delta^2}(1 - 2800\,S).$$

[1]) Cf.: EMDEN, „Gaskugeln", p. 285.

The value of S is still a doubtful point.

If we should assume that the nebula emits black radiation, S would already be $= \frac{1}{2800}$ at a mean temperature of not quite 1500°, hence attraction and repulsion would be about equally great.

This is undoubtedly erroneous; the radiation has been smaller than is calculated on the strength of STEPHAN-BOLTZMANN's law. On the other hand the nebula will have emitted other radiation [1]) besides temperature-radiation, which again partly compensates the deficit.

In our opinion the result of this research may, therefore, be summarized as follows:

On account of the uncertainty which prevails with regard to the quantity of energy emitted by the nebula, it is difficult to make an accurate estimation of the amount by which the attraction of the mother nebula on a newly separated planet must be diminished in virtue of the pressure of radiation. In consequence of the contraction, both of planet and of nebula, the effect in question will continually diminish, and in general it will also have been greater for the larger and more remote planets. Taking everything together it is not excluded that the said diminution, had a quite appreciable amount, at least for the large planets.

If, therefore, in our solar system particularities should occur which can be accounted for as the result of such a change of gravitation, there is every reason to accept this explanation. And this seems actually to be the case, among others with the small inclinations and small eccentricities of the large planets. (See among others NÖLKE loc. cit.). To enter more fully into this, would lead us too far.

§ 4. *Gravitation and pressure of radiation in a nebula.*

Departing from our considerations in the preceding §§ we shall now consider the more irregular nebulae, which present two or more condensations, as e.g. the Dumbbell nebula. Most nebulae have dimensions which are probably to be measured in thousands of Neptune orbit radii. Not much is known about their masses. But when we assume that a multiple star will be formed out of such a nebula, we must assign to each of the parts of the nebula a mass of the same order of magnitude as our sun has. In order to effect a rough estimation of the acting forces, we shall more fully discuss the following system.

[1]) The light of the tails of comets, and probably of most nebulae, arises besides through temperature radiation, through other processes, which are not yet entirely known.

Two nebular spheres, each with a mass of 7.10^{33} gr. and a radius of 200 Neptune orbit radii have centres which are at a distance of 1000 Neptune orbit radii from each other. The density, like the intensity of radiation, is again considered constant inside the nebula.

The quantity P, which occurs in the formulae (5) and (6), may be written:

$$P = 2\mu\varrho_0 R_0 \left(\frac{R_0}{R}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which μ, ϱ_0 and R_0 have the same value as in the preceding §, in which R_0/R is, however, ± 300 times smaller. Then the value of P becomes so small that we may write instead of (5):

$$A = \frac{\pi r_1^2 \cdot \frac{4}{3}\pi R^3 S}{\Delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Assuming that of this quantity of radiation, which one nebula sends to another, the nth part is absorbed by the latter sphere, this experiences a pressure of radiation which may be written after some reduction:

$$D = f\frac{M_1 M_2}{\Delta^2} \cdot \frac{S}{n} \cdot \frac{3R^5}{4fc\varrho_0^2 R_0^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When numerical values are introduced, we get:

$$K = f\frac{M_1 M_2}{\Delta^2}\left(1 - 3.10^{15}\frac{S}{n}\right).$$

Even on the assumption that only $\frac{1}{100.000.000}$ of the received radiation is absorbed, the value of S need not be more than $0{,}3.10^{-7}$ to render the effect of the pressure radiation as great as that of gravitation. In the case of black radiation, a temperature of some tens of degrees above the absolute zero would already suffice to bring it about. The real temperature will on an average certainly be higher, besides in this case considerable luminiscence should also be taken into account, so that we come to the conclusion:

There is every reason to expect that in nebulae with some condensation nuclei the gravitation of the different parts on each other has greatly diminished, if it is not quite exceeded by the mutual pressure of radiation.

For the rest it should be pointed out that this diminution of gravitation only refers to the *internal* gravity. Towards the outside the ordinary Newtonian attraction remains valid.

SUMMARY.

After having adduced some grounds in § 1 why it seems desirable to study the effect of pressure of radiation in nebulae, we examined the *system nebula-star* in § 2. In this case the value of the pressure of radiation remains so small that even under favourable conditions no effect can be expected. Treating in § 3 the *system nebula-planet*, we saw that it is not excluded that in the first time of their formation from solar nebula the large planets were subjected to a strong pressure of radiation, and this may perhaps be responsible for some peculiarities in the system.

In the investigation of the system formed by *two or more nebulae* in § 4 we came to the conclusion that if anywhere, the pressure of radiation must manifest itself here.

Bussum, April 1921.

Astronomy. — *"The Orbit of Bu* 6832 = Σ 1834". By W. H. VAN DEN BOS. (Communicated by Prof. W. DE SITTER). Communication from the Observatory at Leiden.

(Communicated at the meeting of June 25, 1921).

In the present investigation a preliminary orbit is determined of Σ 1834. It is as yet scarcely possible to derive reliable orbital elements for this double star.

The difference in magnitude being very small, the quadrant becomes doubtful about the time of periastron-passage. The same difficulty occurs in the well-known binaries ξ Scorpii and ζ Boötis, the first of which has been investigated by T. N. THIELE (Astr. Nachr. 1199, Bd. 50), the second independently by AITKEN (Publ. Astron. Soc. Pacific, Vol. XXVIII N°. 165) and HERTZSPRUNG (Astr. Nachr. 4871, Bd. 203).

In both cases however the binary character of the motion was certain, whereas in the case of Σ 1834 the assumption of uniform rectilinear motion has been investigated only three years ago by Prof. DOOLITTLE (Astr. Journ. Vol. XXXII Nr. 746), though LEWIS in his volume on the Struve-stars already pointed out the probability of orbital motion. DOOLITTLE finds both curvature of the path and acceleration along the path.

At present there can no longer be any doubt, that the companion is again in the second quadrant. In Vol. XXXI, Nr. 180 of the Publ. Astr. Soc. Pacific Prof. AITKEN gives measures of the pair, confirming the binary character. He says, that in the best two nights the following star seemed to be somewhat fainter. I noticed the same on a splendid night this year. But we must not have too much confidence in such statements. There are however more serious objections against the supposition, that the companion is still in the fourth quadrant at present.

If we plot the position-angles and distances up to 1892 against the time, and derive normal places for 1830, '40, etc. 1890 by a graphical adjustment, these places not only exhibit a distinct curvature and acceleration, but also show the impossibility of placing the recent observations in the third and fourth quadrant. Moreover, if we consult the measures between 1893 and 1914, we see that

the pair must have been very close in 1897 (BRYANT: "round", LEWIS: "possibly elongated"), that there are tolerably concordant measures by BRYANT and AITKEN about 1903, and that the pair is single again in 1904 (HUSSEY) and 1906 (AITKEN). Because of their large discordances the measures between 1893 and 1914 have been omitted in the computation of the orbital elements, though it will be of interest to see how they are represented.

The observed distances, plotted against the time, show symmetry about the epoch 1904·8. Therefore I assumed as a first approximation, that the major axis and the line of nodes are coincident, so that we have: $T' = 1904{\cdot}8$. and $\omega = 180°$. Provisional values for the other elements were derived graphically, the period by measuring the areas of two sectors. These values are:

$$P\ 280\ \text{years}\quad e\ 0{\cdot}83\quad a\ 0''{\cdot}89\quad i \pm 77°6\quad \Omega\ 107\overset{\circ}{,}7.$$

They give already small residuals for the reliable measures, but a marked systematic deviation of the angles between 1866 and 1893 shows, that the assumption $\omega = 180°$ is erroneous. Normal places for the epochs 1830, '40, '50, '60, '70, '77, '84, '90, 1915 and 1921 are derived from interpolation-curves of angles and distances, and a solution by the method of least squares, using COMSTOCK's formulas (The Orbit of Σ 2026, Astr. Journ. Vol. XXXI N°. 725) and giving half weight to the distances, gives the elements:

T	1901·73
P	295·6 years
e	0·823
a	0″93
ω	169°2
i	± 82°04
Ω	110°6

angles increasing.

The measures of the next decade will give a better definition of the present half of the orbit, and consequently a closer approximation of the elements.

The hypothetical parallax, $aP^{-2/3}$, is 0″021. If we adopt the mean value for the sum of the masses, given by AITKEN in The Binary Stars, 1·76 ☉, the parallax will be 0″018 and the mean distance 52 astronomical units. The maximum difference in radial velocity is then 17 km. per second. The spectrum is $F8$ according to the Draper Catalogue, and the photographic magnitude of each component is 8·5. The pair should have been observed with a large spectroscope in the neighbourhood of periastron. The absolute magnitude of each component is $8{\cdot}5 + 5\ log\ 0{\cdot}018 = +0{\cdot}1$, in good agreement with the value to be expected from the spectral-type.

MEASURES AND RESIDUALS (observed — computed).

The columns give respectively the date, the observer, the aperture of the instrument in inches, the observed position-angle, reduced to 1925, the observed distance, the number of nights, remarks by the observer, and finally the residuals in angle and distance.

1829·74	Σ	9·6	113°0	1″36	2		+ 1°4	— 0″01
30·28	h	5	104°0	1″09	3.2		— 7°6	— 0″27
32·32	h	5	112°2	1″20	2.1		+ 0°5	— 0″14
·66	Σ	9·6	113°4	1″37	2		+ 1°7	+ 0″03
40·60	Da	7	111°6 [1])	1″14	2		— 0°5	— 0″11
41 53	OΣ	15	112°6	1″21	2		+ 0°4	— 0″03
43 23	Ma	9·6	113°5	1″37	2.1 [2])		+ 1°2	+ 0″15
48·99	Da	7	111°5	1″07	2		— 1°1	— 0″07
51·51	OΣ	15	113°0	1″00	1		+ 0°2	— 0″11
57·57	Se	9	114°5	0″92	2		+ 1°3	— 0″09
66·44	Δ	7	113°0	0″87	3.2		— 1°1	+ 0″01
·49	Tal.	10 [3])	110°6	0″87	1		— 3°5	+ 0″01
71·21	Du	9·6	115°2	0″68	4		+ 0°5	— 0″07
·53	Gl	9·3	115°0	0″6 (tax.)	1	uncertain	+ 0°3	— 0″15±
72·54	OΣ	15	116°2	0″95	1		+ 1°3	+ 0″22
874·42	WS	8¼	113°4	0″6 (tax.)	3		— 1°8	— 0″09±
75·48	OΣ	15	115°4	0″65	1		0°0	— 0″01
79·47	Hl	26	114°6	0″46	3		— 1°7	— 0″11
80·94	Big	12 [4])	116°8	0″47	2		+ 0°1	— 0″06
81·53	Smith	8¼ [5])	124°3	0″6 (tax.)	1		+ 7°4	+ 0″09±
·54	H.Pt.	12¼ [4])	119°5	0″59	1		+ 2°6	+ 0″08
·55	Sk	8¼	124°3	0″5 (tax.)	2.1		+ 7.4	— 0″01±
83·61	En	7½	117°2	0″51	6		— 0°4	+ 0″06
84·70	Sk	8¼	112°4		2.0	doubtful, very close	— 5°9	
85·53	Hl	26	118°4	0″42	3		— 0°1	+ 0″02
92·17	β	36	124°9	0″26	2		— 1°0	+ 0″08
93·47	Lew	12¾	145°3	0″39	1		+ 15°0	+ 0″22
·58	Com	15½	113°4	0″25 (tax.)	1		— 16°9	+ 0″08±
95·53	„	15½	single					

1897·51	Lew	28	may be elongated at		166°3		− 82°2	
·63	B	28	round			comp. dist. 0″06		
99·40	Brown	26	270°±	0″20±	1		− 8°7±	+ 0″09±
·45	B	28	200°	elongated	1		− 78°7	
1900·49 [6])	B	28	209°1	0″22	1		− 75°2	+ 0″08
·54 [6])	Bow	28	206°7	0″25	1		− 77°6	+ 0″11
1·16	Lew	28	201°9	0″23	1		− 85°8	+ 0″07
·31	B	28	219°3	0″18	3	not separated	− 68°4	+ 0″02
·50	Lew	28	178°9	0″1 (tax.)	1		+ 71°2	− 0″06±
2·73	B	28	298°1	0″29	1	elongated	+ 6°0	+ 0″12
3·60	A	36	276°2	0″12	1		− 18°5	− 0″04
4·34	Hu	36	round		1	comp. dist. 0″15		
6·43	B	28	2°1	0″14	1	elongation very slight	+ 54°7	+ 0″03
·61	A	36	no elongation		1	comp. dist. 0″11		
7·32	B	28	343°1	0″13	2	slightly elongated	+ 27°9	+ 0″04
8·38	Do	7¼	52°5	0″35 (tax.)	1		+ 79°3	+ 0″29
·51	Do	7¼	14°5	0″30 (tax.)	1		+ 41°3	+ 0″24
9·59	B	28	may be elongated at		110°		+105°8	
·60	B	28	108°6	0″09	1	definition bad, doubtful	+104°4	+ 0″04
1909·76	B	28	104°3	0″15	1	not separated	+100°1	+ 0″10
10·28	B	28	98°6	0″22	2		+ 71°4	+ 0″17
11·43	B	28	102°1	0″16	3		+ 44°0	+ 0″09
·49	Bow	28	61°0	0″19	2		+ 2°9	+ 0″12
·49	A	36	83°7	0″17	2		+ 25°6	+ 0″10
12·46	B	28	91°0	0″16	3		+ 19°8	+ 0″07
·53	Bies	15	65°?	<0″25	1	plutot simple	− 6°2	<+ 0″16
14·50	Gr.Obs.	28 [8])	102°9	0″15	2		+ 18°7	+ 0″01
14·58	A	36	85°9	0″22	2	good	+ 1°3	+ 0″07
15·44	Gr.Obs.	28 [8])	97°4	0″20	3		+ 9°9	+ 0″03
16·38	A	36	87°0	0″28	2	good	− 3°0	+ 0″09
19·62	Gr.Obs.	28 [8])	94°9	0″28	1		− 0°1	+ 0″00
21·36	v.d.Bos	10½	92°3	0″37 (tax.)	2 [7])		− 4°3	+ 0″05
·41	Gr.Obs.	28 [8])	98°2	0″39	3		− 1°5	+ 0″07

[1]) I reject a single angle, noted by DAWES: "very bad".

[2]) DOOLITTLE gives: 3 nights. MAEDLER gives: 2 nights in "Untersuchungen über die Fixstern-systeme", but in the Dorpat Observations there is but a single position-angle.

[3]) DOOLITTLE gives a measure by BARCLAY. This is probably an error, for a correction of precisely 2 years in the time and of 10° in the angle makes it identical with TALMAGE's measure, which is given in "Leyton Observations", but noted BARCLAY there (DAWES' measures are called "Bishop").

[4]) Not in DOOLITTLE's list.

[5]) Called "Seabroke" by DOOLITTLE.

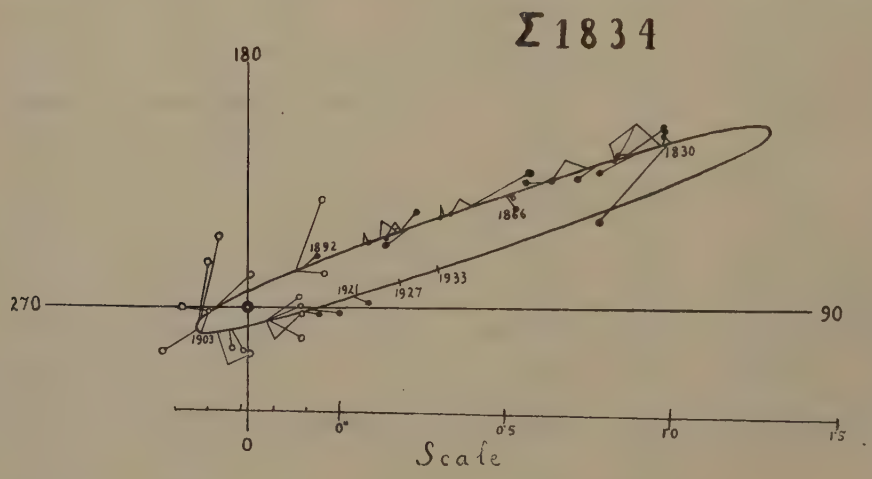
[6]) Given by LEWIS in Mem. R. A. S. LVI; I could not find them, neither in the Monthly Notices, nor in the Greenwich Observations.

[7]) My separate results are

1921·341	92°5	0″35 (tax) splendid definition.
·379	92°1	0″40 (tax.) definition good.

[8]) These measures have been communicated to me by Mr. JACKSON; they were received after the computation had been finished, but they are very well represented. The separate angles for 1921 are 93°3, 100°0 and 101°3, fairly indicating the degree of uncertainty adhering to the measures even now.

My search for other recent measures has unfortunately been in vain. Up to 1893 the measures are well represented. Only in 1914



the observations become reliable again. Even in 1911 AITKEN's measures on two nights differ by 7°4 in angle, and the observations by BRYANT and BOWYER at the same time differ by about 20° from AITKEN's. It is probable, that a large part of the measures between 1893 and 1914 are merely optical illusions. In the diagram the measures used in the computation are given by dots, the others by white circles, and every measure is joined with the computed place by a thin line.

	Ephemeris.	
1923·50	98°2	0″37
25·50	99°2	0″42
27·50	100°1	0″46
29·50	100°8	0″51
31·50	101°4	0″55
33·50	101°9	0″59

Leiden, Observatory, 1921 June 23.

Physiology. — *"Muscle Sound in Birds"*. By Miss L. KAISER. (Communicated by Prof. G. VAN RIJNBERK).

(Communicated at the meeting of April 30, 1921).

Concerning muscle sound and action currents in birds I found no data except a provisional communication by WEISS [1]), which as far as I know has not been followed by a more ample one. WEISS recorded the sound of the m. pectoralis of a pigeon by means of a very thin membrane, the pigeon being poisoned with strychnine. He so obtained a curve in which the distance between two crests varied from $\frac{1}{80}$ to $\frac{1}{165}$ second. The action current of the same muscle in similar conditions showed waves of a duration of $\frac{1}{85}$ to $\frac{1}{170}$ second, which agrees perfectly with the values mentioned above. SCHWARZKOPF [2]) found, as other investigators (MAREY, RICHET, and EXNER) had found before him, that the number of stimuli necessary to produce complete tetanus for pigeon muscle is 70 to 125 per second. PÜTTER [3]) mentions 70 to 80 per second as the greatest number of separate contractions possible in bird's muscle.

EWALD [4]) has described a vibration, which he observed in pigeons and in a few other birds at the upper edge of the orbita and at the eyeball. He supposes a simultaneous contraction of both the m. obliqui to be the cause of this vibration and calls the phenomenon "Augenschwingen". By means of a lever, attached to the head of the pigeon and resting upon the upper edge of the orbita, as well as by means of a double needle with writing apparatus fixed into the eyeball, EWALD succeeded in recording this phenomenon. The curves obtained by these two methods practically confirm each other, the rate of the waves being 25 to 30 per second.

The same phenomenon was further examined by the author.

The vibration, which may be observed by touch, over the whole head of the bird, is also very evident to the ear. A phonendoscope placed upon the head of the pigeon conducts to the ear a low sound, in which a tone of a low pitch may be distinguished.

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[1]) Centralblatt f. Physiol. Bd. 26. 1912.

[2]) Pflüger's Archiv. Bd. 121. 1908.

[3]) A. PÜTTER, Vergleichende Physiologie, 1912.

[4]) Archiv. f. experiment. Pathol. u. Pharmakol. Suppl. 1908. Festschrift SCHMIEDEBERG.

Even without any apparatus, by holding the head of the bird to the ear, the sound may be perceived sufficiently.

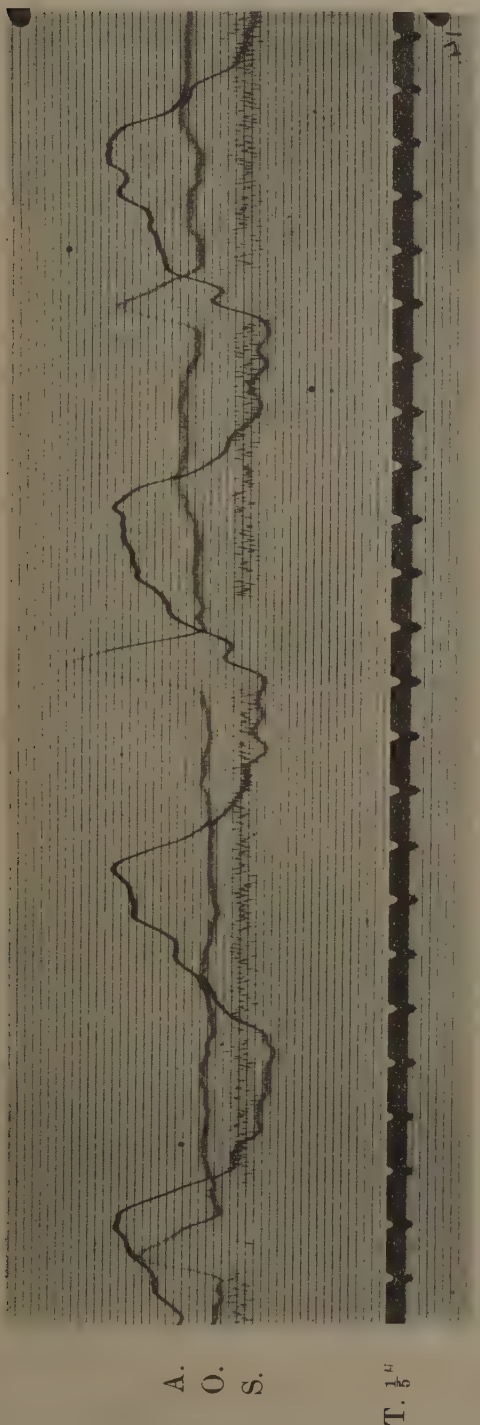


Figure 1.

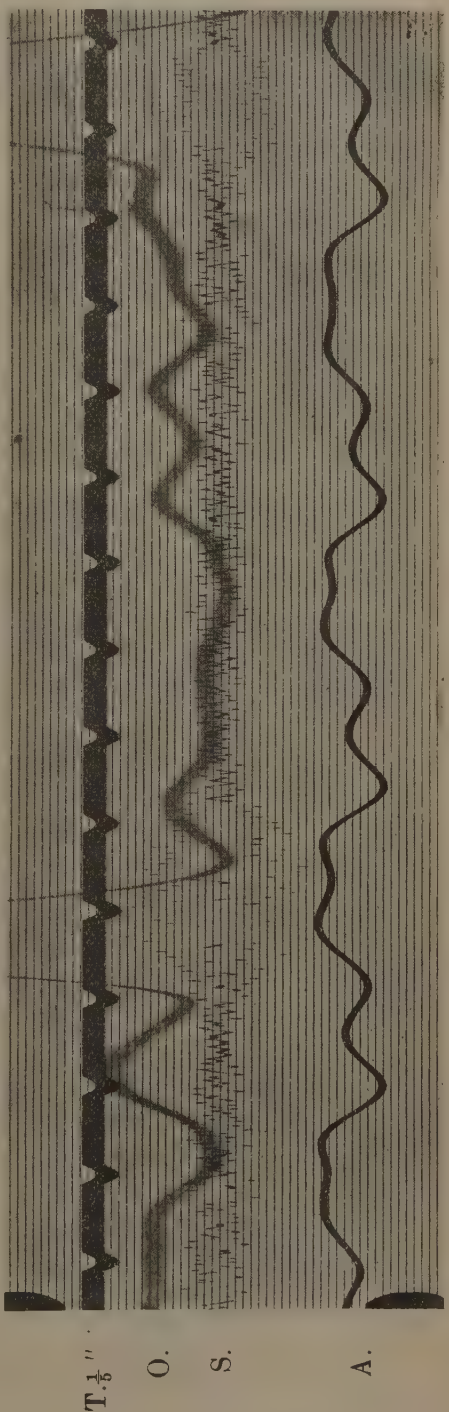


Figure 2.

These curves were obtained by conducting the sound by means of a microphone to the string of the string galvanometer (Fig. 1 and 2).

Besides the movement of the string, *S*, the respiratory movement, *A*, and the movement of the upper eyelid, *O*, were recorded. In these figures appears a distinct relation existing between the sound and the movement of the eyelid, whereas no constant relation between the sound and the respiratory movement may be detected. As will be exposed in the Archives Néerlandaises de Physiologie, the sound is caused probably by a simultaneous contraction of all extrinsic eyemuscles, which coincides with the going down of the upper eyelid. As may be seen in the figures the curve of the string was regular and distinct. The rate of the vibration was always about 17 in one fifth second, that is about 85 per second.

Apart from the fact that there is no agreement between the different authors concerning the relation between the pitch of the muscle tone and the number of stimuli applied to the muscle nor concerning the number of stimuli necessary to produce complete tetanus and the number of impulses in voluntary contraction, in this case the number 85 agrees very well with the statements of SCHWARZKOPF and PÜTTER concerning the number of stimuli needed by bird's muscle to go into tetanus and also with the values found by WEISS in the case of contraction by strychnine.

Physiology. — "*A Simple Method to obtain a Curve of the Contraction of the M. arrectores pilorum*" [1]). By Miss L. KAISER. (Communicated by Prof. G. VAN RIJNBERK).

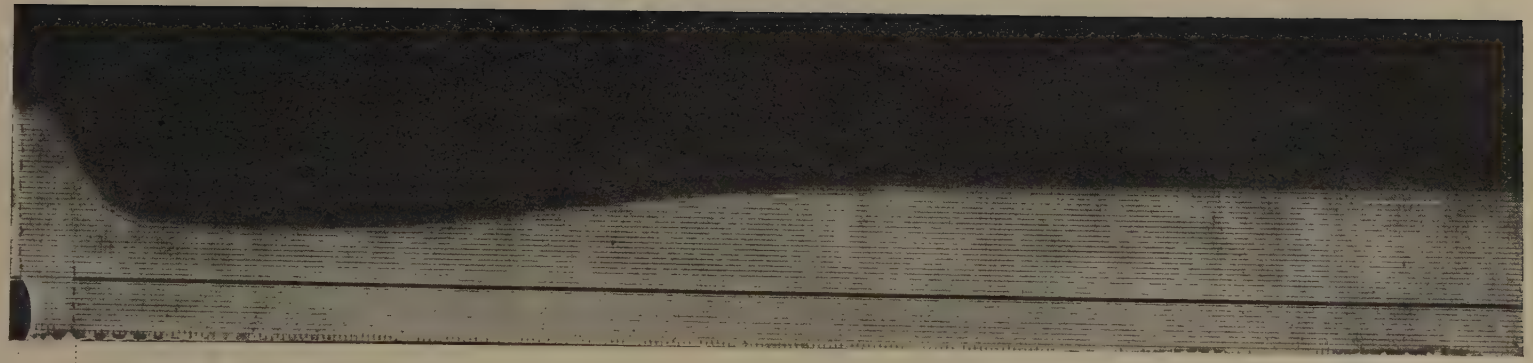
(Communicated at the meeting of June 25, 1921).

When placing the tail of a cat before the slit of a camera with a vertically moving plate, so that the dorsal side of the tail is directed towards the middle of the slit, we obtain on the sensitive plate a rather well-defined shadow. When stimulating the sympathetic chain, by which process the hairs are elevated, this elevation is imaged on the plate as a broadening of the shadow. The outer border of this shadow being, indeed, a rather sharp line, it affords a curve of the contraction of the M. arrectores pilorum, illustrating various features of this contraction, such as duration of the latent period, duration and shape of the crescent, etc. as the figure shows.

By this method I made several records. In all cases the sympathetic chain was stimulated with the tetanic current of an ordinary induction-coil. The curve, then, represents a tetanic contraction and not a simple one. We ascertained the average duration of the latent period, and its variations by altered strength of the stimulus, by fatigue, etc. We also determined the duration and the steepness of the ascending part of the curve under different conditions. Finally also the height of the contraction was estimated.

From this height the real shortening of the M. arrectores pilorum might be derived. Mr. WOERDEMAN furnished me with some necessary data, which he found by measurements in a microscopic preparation of a cat's skin. Let the distance from the insertion of the M. arrectores into the hair to the turning point of the latter be 400 μ, and let the length of a hair be $2\frac{1}{2}$ cm., then, at the extremity of the hair the magnification of the contraction will be $\times$ 60. The projection caused an additional enlargement of $\times$ 1.2, so that the entire magnification must be fixed at about $\times$ 75. An average broadening of the shadow of 2,5 cm, as found in my curves, points, therefore, to a shortening of the muscles of 250 μ.

[1]) After experiments made in the Laboratory for Physiology of the Amsterdam University.



S = the shadow of the tail retaining its original breadth for some time after the beginning of the stimulation (latent period) then broadening rather rapidly, in order to narrow again very slowly.

P = sign marking the stimulation.

T = the time in $\frac{1}{5}$ seconds.

This shortening amounts to about half the length of the muscle in rest, as found in the above preparation of the M. arrectores.

It goes without saying that with this method, as well as by writing a contraction curve on a rotating drum, we obtain an imperfect image of what really takes place. The error made in this way is easily reduced to a minimal value, and is moreover easy of calculation, so that the true curves can be deduced from those obtained.

Since, to my knowledge, LEWANDOWSKY [1]) is the only researcher who has written curves of a smooth muscle in warm-blooded animals, contracting by indirect stimulation, it seems to me that valuable data may be obtained by using the method here described.

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[1]) DU BOIS REYMOND's Archiv. 1899.

Physiology. — *"Regarding Automatic Movements of the Isolated Iris"* [1]. By JASPER TEN CATE. (Communicated by Prof. G. VAN RIJNBERK).

(Communicated at the meeting of June. 25, 1921).

After SERTOLI's description of automatic rhythmic movements of the M. retractor penis (1882), numerous other organs with nonstriated muscle-cells have been investigated with regard to this phenomenon. It has been proved that the stomach, the guts, the ureters, the uterus, the arteries, the spleen, the bladder, the gall-bladder, and the oesophagus, in cold- as well as in warm-blooded animals, exhibit rhythmic contractions when surviving outside the body under favourable circumstances. In his researches MAGNUS has demonstrated that, as regards the gut, this faculty resides in the local nervous apparatus of AUERBACH's plexus. Since all the organs mentioned also possess local nervous plexus, it seems likely that also in the case of these organs the rhythmic contractions are attributable to an automatic function of those local nervous apparatus. Besides our knowledge of these facts justifies the assumption that other organs, provided with smooth muscle-cells, local nervous plexus and ganglia, are also capable of performing automatic rhythmic contractions. This induced me to experiment on the iris. This organ with an abundance of smooth muscles, also possesses a well-developed apparatus of ganglia, as has been made out by the uniform results of the most recent histological inquiries (LAUBER 1908, SCHOCK 1910, POLLACK 1913). There was good reason to suppose, therefore, that also the isolated iris should be capable of executing automatic rhythmic movements. However, the muscles of the iris being very feeble we had to cast about for a technique which, with least friction, should permit a registration of the result as much enlarged as possible.

With this view I made the following contrivance: A blade of straw was fixed to a glass bar for a lever. The fulcrum was constituted by a thin slightly twisted silk thread attached to the two prongs of a glass fork. The iris pulled at the end of the glass lever. The magnification of the displacement of the free end of the straw

[1]) After experiments made in the Physiological Laboratory of the Amsterdam University.

was × 16. Registration took place by arranging the contrivance before the slit of a camera with vertically moving plate, in the pencil of light emitted by a projection lantern.

I experimented chiefly on the cat's iris. The eye was rapidly enucleated and preserved in Tyrode-solution of 37—38° C. Glucose soon appeared to have an unfavourable action, so that tyrode was used without glucose. When oxygen was administered the isolated iris shortened considerably, but the spontaneous movements were much feebler than without the perfusion of O^2. In the latter case the iris relaxed and spontaneous contractions soon followed. They were stronger but of shorter duration than when oxygen had been supplied. I found the optimal temperature to be 37°—38° C. All these conditions being fulfilled, it was not difficult to establish two sorts of movements in the iris-preparation. First: movements that were comparatively strong, very slow and apparently analogous to those known in the literature as "tonus-oscillations". Secondly: movements that were much weaker and more frequent, bearing a distinct resemblance to what are generally termed: "spontaneous rhythmic movements" of isolated organs. They were not exactly regular, as is the case with other organs: their rhythm, as far as we could ascertain, varied in normal relations, in Tyrode, from 16 to 29 contractions per minute. Nevertheless our findings have established the capacity of the isolated iris of executing spontaneous, automatic, rhythmic contractions.

Furthermore, I have dealt with a few pharmacological problems viz. that pilocarpin and cholin reinforce these movements as well as the tonus-oscillations, whereas adrenalin weakens them and atropin inhibits and ultimately abolishes them. Under the influence of pilocarpin the rhythm is accelerated up to 25—38 contractions per minute. In the presence of adrenalin the frequency falls to 4—18.

Chemistry. — *"The Electromotive Behaviour of Aluminium"*. III. [1]). By Prof. A. SMITS and C. J. DE GRUYTER. (Communicated by Prof. P. ZEEMAN).

(Communicated at the meeting of May 28, 1921).

1. In a previous communication on the same subject we already pointed out that the *E-X*-diagram of the four-component system Al'''—Hg''—Anion—H_2O must be classified among the type in which the potential corresponding to the three-phase equilibrium, lies between the potentials of the two metals. Hence the potential of aluminium in an Al-salt solution might be expected to become less negative on addition of a mercury salt, whereas it was the reverse that was found. Already at the beginning of the study on the electromotive behaviour of aluminium we expressed the opinion that this remarkable phenomenon shows that as a rule the pure aluminium is passive, i.e. is greatly disturbed in noble direction, and that small quantities of mercury, dissolved in aluminium, exert a positive catalytic influence on the establishment of the internal equilibrium, so that the disturbance disappears.

In order to examine the influence of mercury on the aluminium potential more closely, measurements were carried out, in which the aluminium electrode was placed in an aqueous aluminium salt solution, to which a mercury salt solution was added at intervals. It should still be stated that to prevent the action of air-oxygen, the experiment was made in a nitrogen atmosphere, the solution being vigorously stirred.

Then the interesting result was found that at first the potential of the aluminium became less negative, then reached a minimum, after which it rose to a strongly negative value, and at last a maximum appeared. Finally a great decrease was observed, during which the potential descended to near that of mercury.

To past the just-mentioned maximum the aluminium electrode was covered with a greyish precipitate, but during the last stage the aluminium electrode was mercurized to a shiny surface.

Before proceeding to the further discussion of the study of this

[1]) These Proc. Vol. XXII, N°. 9 and 10, p. 876.
" " Vol. XXIII, N°. 7, p. 966.

phenomenon it seems desirable first to indicate the significance of this course of the potential.

2. At first the aluminium electrode was immersed in the aqueous solution of $Al_2(SO_4)_3$, and then a solution of $HgCl_2$ was added. Directly after this addition aluminium goes into solution, and mercury is deposited, which will have been preceded by the discharge of $Hg^{\cdot\cdot}$ to $Hg^{\cdot\cdot}_2$.

As has been said the equilibrium-E,X-diagram of the system Al-Hg-electrolyte must belong to the type in which the potential of the three-phase equilibrium Al-phase-electrolyte-Hg-phase lies between the potentials of the two metals (in internal equilibrium), i.e. on addition of a mercury salt the solubility product of the aluminium decreases more greatly in the formula:

$$E = -\frac{0.058}{3} log \frac{L_{Al}}{(Al^{\cdot\cdot\cdot}_L)} - 2{,}8$$

in consequence of mercury dissolving in the aluminium, than the relative concentration of the aluminium-ions in solution, which remains even practically constant.

Now it is clear that a certain time is required for the solution of mercury in the aluminium, and that in the very first moments there will be pure aluminium, on which mercury has precipitated.

Hence at first L_{Al} will not have been changed by dissolved mercury, and as also $(Al^{\cdot\cdot\cdot}_L)$ has practically remained the same, it might be expected that in the beginning E does not change. In this reasoning an important factor has, however, been overlooked, viz. the fact that the aluminium is attacked! Electrons and ions are withdrawn from this metal with great rapidity when mercury is precipitated, and non-activated aluminium being a very inert metal, disturbance in noble direction can then occur. But this disturbance can only be observed for a short time, for the precipitated mercury will soon dissolve somewhat in the aluminium, and give rise to activation, which according to the more recent view means a conversion in the direction of the internal equilibrium.

Hence a change of the potential in noble direction is followed by a minimum and a great change in base direction. When once the aluminium has been activated, and has more or less reached the state of internal equilibrium, the equilibrium between the stable mixed crystal phase, the liquid mercury phase, and the electrolyte in the boundary layer will soon have been established, and the potential will remain constant for some time.

As, however, the electrolyte in the boundary layer practically

contains no mercury, but the solution outside the boundary layer does, the precipitation of mercury will continue, and at last the mixed crystal phase *s* in the surface will have been entirely converted to the mercury *l* (see Fig. 2). At this moment the electrode will be perfectly shiny, but the deposition of mercury has not stopped yet, so that now a fall of the potential of the liquid mercury phase will occur, till the electrolyte and the liquid mercury phase are in equilibrium, or in other words, the process continues till practically all the mercury from the electrolyte has been precipitated, and the end-potential will have approached the mercury potential the more as the surface liquid phase contains less aluminium.

In order to represent this interesting change of the electrical potential of aluminium through mercury in the most striking way, we have first modified the experiment somewhat, and then registered the phenomenon photographically.

3. The modification in the experimentation consists in this that the aluminium electrode was directly immersed in an aqueous solution of an aluminium salt, to which a little of a mercury salt was added. This solution was brought into contact with the same solution, but without mercury salt by means of a siphon, and this solution also contained an aluminium electrode, so that the potential of the experimental electrode was measured with respect to aluminium. Pure nitrogen was led through both solutions, which prevented the very troublesome disturbance which always accompanies attack by oxygen.

Fig. 1 gives a clear representation of the first part of the phenomenon. The line *ab* indicates the position of the light image, when

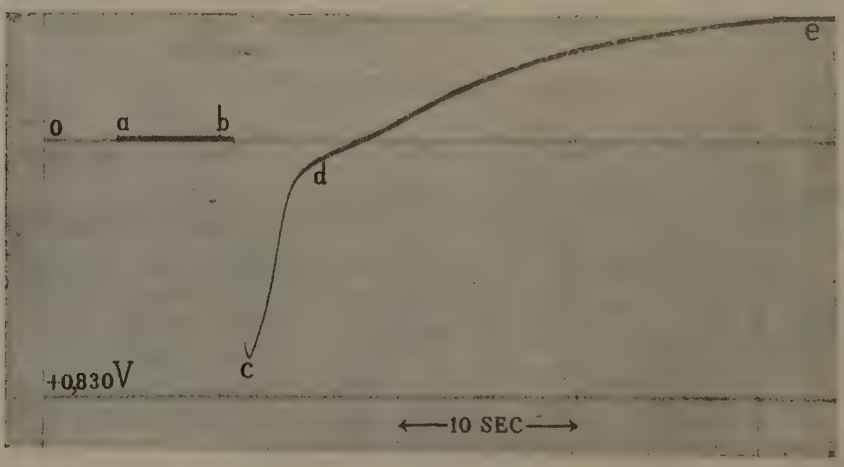


Fig. 1.

the aluminium cell is not yet closed, or what comes to the same thing, when the aluminium experimental electrode is immersed in the same pure aluminium sulphate solution in which the aluminium measuring electrode is placed.

In *b* the experimental electrode was suddenly immersed in the solution of aluminium sulphate + sublimate. On being attacked the potential of the experimental electrode is immediately greatly disturbed. The amount of this disturbance may be inferred from the place of the gauging line, which corresponds with a potential difference of + 0.830 Volt.

In consequence of the evidently strong catalytic action of the mercury dissolved in the metal surface, this disturbance not only rapidly diminished in *c*, but the potential rises to a much more negative value than that of the aluminium in the initial state. This shows, therefore, that the aluminium has passed into a much less noble and more active state through the dissolved mercury, which means in other words that the mercury here enormously accelerates the conversion in the direction of the internal equilibrium, notwithstanding the fact that under the given circumstances this conversion is attended with a stronger attack. At *e* the potential reaches its most negative value, and remains there constant for some time. As the aluminium is here covered with finely divided mercury, which gives the electrode a greyish colour, this constant potential, when at least the same state prevails throughout the metal surface, corresponds with the three-phase equilibrium *c*, *s*, *l*, i. e. with the heterogeneous equilibrium between the activated mercury-containing aluminium mixed crystal phase, the aluminium-containing liquid mercury phase, and the electrolyte in the boundary layer.

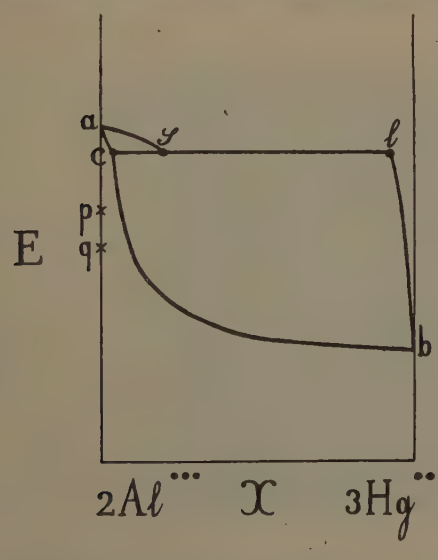


Fig. 2.

Accordingly summarizing we may say that when we start from a potential at the same level as the point *p*, this potential at first descends to a value at the level of e. g. point *q*.

When activation has taken place, and the three-phase equilibrium has been established, the potential is indicated by the line *c*, *s*, *l*, which lies much higher. In virtue of the mercury content of the

aluminium mixed-crystal phase, which at the ordinary temperature is only a few atom percentages, and in virtue of the fact that the electrolyte c practically contains no mercury, it may be expected that the line s, l, c is situated only exceedingly little lower than the point a. But when as here in these investigations the electrolyte is a solution on which the activated aluminium acts, and the metal is, therefore, attacked with separation of mercury, the potential will yet be found too little negative either through disturbance or through change of the concentration of the metal phases, unless the state of formation of the hydrogen could annul this effect.

As, however, the coexisting electrolyte of the three-phase equilibrium lies certainly very strongly one-sided on the aluminium side in the system Al-H-electrolyte, the state of formation of the hydrogen can here exert no appreciable influence on the potential. Our conclusion is, therefore, that the observed maximum negative potential will be less strongly negative than that of the pure aluminium in the state of internal equilibrium. If the electrode has already been entirely mercurized locally, the potential will of course be found much too little negative. The most negative potential was found by immersion of an Al-electrode activated with mercury in a pure $Al_2(SO_4)_3$-solution. This was —1,58 with respect to the 1N calomel electrode or $E_H = -1{,}29$. Pure aluminium is always disturbed in noble direction, and to this we owe the usefulness of this metal for all kinds of technical and domestic purposes. Pure aluminium in the state of internal equilibrium would, indeed, be as unsuitable for these purposes as magnesium and calcium.

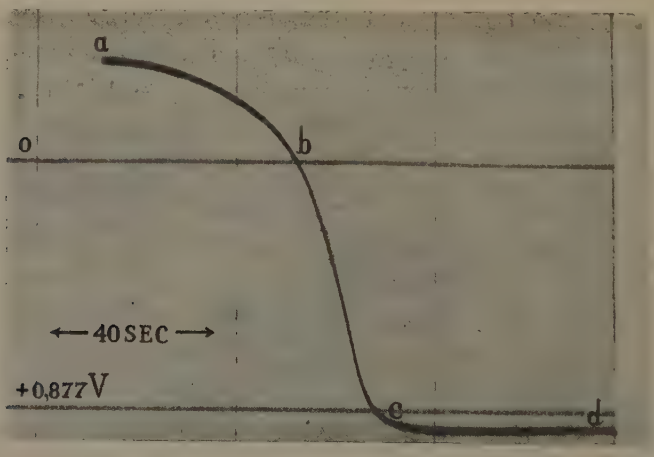


Fig. 3.

The further course of the potential after the maximum negative

value has been reached, is represented on the photo, Fig. 3. As the subsequent process is much slower, the velocity of revolution of the drum of the registering apparatus was chosen smaller here. We see that the potential remains constant for some time, and then proceeds to positive values with ever increasing rapidity. The initial point of this change coincides with the moment at which the grey mercury deposition makes room for the liquid shiny mercury phase.

In a following communication we shall, among other things, state the results obtained when other aluminiums and mercuri-salt solutions were used.

Laboratory of General and Anorganic Chemistry of the University.

Amsterdam, May 8th 1921.

Chemistry. — *"On the behaviour of Amorphous Carbon and Sulphur at High Temperatures and on Carbon-Sulphides"*. By Dr. J. P. WIBAUT. (Communicated by Prof. A. F. HOLLEMAN).

(Communicated at the meeting of May 28, 1921).

§ 1. *Introduction*. In 1919 Dr. A. STOFFEL and the author of this paper published an inquiry into the sulphurous compounds of coal [1]). The result was briefly as follows:

A method was elaborated to determine the sulphur combined with iron (pyritic sulphur) and the sulphur present as organic compounds separately. It was then examined how these anorganic and organic sulphur compounds dissociate during the coking of coal, i.e. the heating with exclusion of air at temperatures of 1000° and higher. It then appeared among other things that during the coking the organic sulphur compounds partly yield sulphuretted hydrogen and volatile organic sulphur compounds, but that a large proportion of the organically bound sulphur from the coal is retained in the coke in the form of a sulphur-carbon compound, which does not lose its sulphur content at the temperature of 1000°. In gas coke, which mostly contains 1—1,5 % of sulphur, this sulphur appeared to be present for the greater part in the form of a carbon-compound, and only for a smaller part to be fixed by the anorganic components (ash components) of the coke. Comparative experiments on the coking of ash-containing coal-samples and of coal-samples that had been freed from mineral admixtures (ash), taught that during the coking of coal, part of the sulphur which is combined with iron as pyrite in the coal, is fixed by carbon in the coke. It seems, therefore, that carbon can combine with sulphur in some way or other at high temperature.

This find was very surprising, and not devoid of importance for the technics of coke-manufacture. About the same time PARR and POWELL [2]) published a research on the same subject, which did not appear as a magazine article, and did not come under our notice until later. The investigators followed another method of research; their results on the whole agree with ours. In two recent papers

[1]) Rec. trav. chim. **38**, 132 (1919).

[2]) A Study of the Forms in which Sulfur occurs in Coal. University of Illinois Bulletin. Vol. XVI. N°. 34 (1919).

POWELL [1]) communicates the result of further researches. In this the behaviour of the sulphur compounds during coking is examined in details for different kinds of coal, in which the already mentioned results were confirmed and extended.

It seems, however, to me that also questions of more general importance present themselves in connection with these reactions. In what way is this compound of sulphur and carbon formed, in which evidently a complex is formed that is very resistent with regard to heating? A preliminary experiment had already taught me that through quick heating of sugar carbon with sulphur a carbon-like sulphurous substance is obtained, which can be made red-hot without losing its sulphur content. Accordingly the mutual behaviour of two simple substances as sulphur and carbon is not yet known in detail. Hence I have begun an investigation purposing to examine the behaviour of amorphous carbon and sulphur on heating, and study more closely the products that arise from these two components. Though this investigation is not yet entirely completed, I feel obliged in view of POWELL's publications, to communicate already now the experiments carried out by me.

§ 2. In order to obtain reproducible results, it was desirable to experiment with an amorphous carbon as pure as possible, which was obtained in a well-defined way.

Finely powdered sugar-carbon obtained through moderate heating of sugar, was extracted with boiling hydrochloric acid, after which the ash-content was 0.30 %. Through extraction with hydrofluoric acid this ash-content is only little diminished. This carbon was heated at 900—1000° in a porcelain tube for 7 hours. The generated gases were pumped off by means of an oil-pump, in a vacuum of ± 1,7 mm. This preparation was analysed, and then again heated at 970—1020° for three hours in a vacuum of 0.6 mm.; during the last hour of this heating experiment there was no generation of gases any more. The evacuation was continued during the cooling. The analysis of this carbon was performed as follows: a weighed quantity of substance was heated in a porcelain boat at 400—450° and 1 mm. for one hour; after having been cooled in vacuum, the boat was quickly placed in a weighing bottle, reweighed, and at once conveyed to the combustion tube. Such amorphous carbon is very hygroscopic; the content of absorbed water determined in this way, was 1.83 %. The analysis of this carbon dried in vacuum

[1]) Journ. Ind. and Engin. Chem. **12**, p. 1069, 1077 (1920).

at 400° showed 98,35 % C. 0,75 % H., and 0,30 % ash. After one hour heating at 600—660° at 2 mm.: 98,68 % carbon, 0,53 % hydrogen, 0,30 % ash. When the rest is estimated as oxygen, and when this content of oxygen is attributed to absorbed water, which has not been removed in spite of the heating at 600° in vacuum, the dried preparation has the following composition: ash 0,30 %, carbon 98,68 %, absorbed water 0,49 %, hydrogen (chemically bound) 0,48 %.

Even after prolonged heating at 1000° in vacuum a small quantity of hydrogen is retained in this preparation. This hydrogen content lies, however, near the limit of the errors of the analysis. Apart from the ash-content, the preparation consists, therefore, practically of amorphous carbon. The ash almost entirely dissolved in hydrochloric acid, and contained but very little iron-oxide.

The following experiments were made with this amorphous carbon K_1: 24 grammes of K_1 were mixed with 8 grammes of pure sulphur; the mixture was placed in a porcelain tube, which was shoved horizontally into an oven heated at 510° C. The temperature of the oven was raised in an hour to 760°, and in the following 90 minutes to 975°. From 600° to the end of the experiment there developed some sulphuretted hydrogen. After cooling much sulphur was found in the colder part of the tube, which had been condensed there. The black carbon-like mass was again placed in a porcelain tube, and this was slowly heated in a vertical oven. At 800° very little sulphuretted hydrogen began to develop; the temperature was carried up to 1000° in two hours, and kept at 1000° for half an hour, then the H_2S-generation had ceased and the heating was discontinued. A sulphur determination in the carbon-like powder showed: 1,98 % S.

16 grammes of P_1 were extracted for a long time with boiling toluene; after evaporation of the toluene no sulphur remained behind. 2,03 % was found for the sulphur content of the extracted product. Hence extraction with toluene does not reduce the sulphur content of P_1. Extraction with carbon disulphide did not lead to the purpose, as it appeared impossible to remove the last rests of carbon disulphide from the extracted product.

I then tried whether through heating in vacuum, in which the receiver was cooled with liquid air to condense gaseous compounds that might possibly be formed, a sulphurous substance could be isolated from this preparation. It appeared, however, in several experiments that even prolonged heating in vacuum reduced the sulphur-content of P_1 hardly perceptibly; hence a volatile sulphur

compound is not formed. The following figures give an idea of the course of such an experiment.

A porcelain boat with 3,09 grammes of P_1 was placed in a porcelain tube, which was open on one side, and was connected there with the oil-pump. The tube was heated in an electrical oven (thermo-element on the outside of the tube).

Time	Temp.	Pressure	
1.30	620°	1.5 mm.	There is formed a little sulphuretted hydrogen.
3.15	840	1.7 „	
3.45	910	1.5 „	
5.10	1010	2 „	
5.45	900	2 „	

Left to cool with evacuation; the sulphur-content of the thus obtained product P_2 was 1.94 %.

2.38 grammes of P_2 heated anew in vacuum:

11.15	400°	1 mm.	
2	940	1 „	
3	980	1 „	
4	1030	1.5 mm.	there still arise traces of H_2S.
5	1060	1.5 „	
6.10	990	1 „	

Left to cool with evacuation. The product P_3 thus obtained contains 1,87 % of sulphur.

It appears from these experiments that the sulphur content of P_1 decreases very little by prolonged heating at about 1000° C. and 1.5 mm., the decrease of the last experiment lies near the limit of the errors of the analysis. The total analysis of the preparation P_1, which had been dried at 600° and 2 mm. for an hour gave: 0.27 % ash, 96.1 % C, 0.33 % H, 2.00 % sulphur, together 98.70 %.

Besides absorbed water this preparation contains, therefore, still a hardly appreciable quantity of hydrogen.

§ 3. Behaviour towards oxidizers and towards hydrogen. In order to get a better insight in the nature of this compound of carbon and sulphur the behaviour of this preparation towards oxidizing and towards reducing agents was examined.

2 grammes of P_1 were shaken with 100 cc. of water and 3 cc. of bromine for four hours on the shaking apparatus at the ordinary temperature. After this operation 3.6 mgr. of sulphur was oxidized

to sulphuric acid, hence 9 % of the sulphur present in the 2 grammes of P_1. In a duplicate experiment 13.5 % of the sulphur was oxidized. In this product the sulphur was, therefore, oxidized only for a small part or at least very slowly by bromine water at the ordinary temperature. When on the other hand a mixture of charcoal with 2 % sulphur was treated in the same way with bromine, the sulphur present was quantitatively found back as sulphuric acid, as was to be expected.

Behaviour towards hydrogen.

In a preliminary experiment 0.90 gramme of P_1 were spread out in a thin layer in a porcelain boat, and slowly heated in a current of pure dry hydrogen. Up to 500° no formation of sulphuretted hydrogen could be found; this began at about 550°. In two hours the temperature was raised from 550 to 750°; in this temperature range sulphuretted hydrogen was slowly but regularly developed. About $^1/_3$ of the sulphur originally present in the P_1 was converted to sulphuretted hydrogen.

In a second experiment 2 grammes of P_1 were heated in a hydrogen current; at 430° sulphuretted hydrogen began to evolve. When 650° was reached, this temperature was kept constant, till the regular generation of sulphuretted hydrogen diminished. The heating at 650° had then been continued for 5 hours. The temperature was then raised to 750°, which gave rise to the formation of some more sulphuretted hydrogen. After the heating at 750° had been continued for four hours, only very little sulphuretted hydrogen was slowly developed, after which the experiment was stopped. The quantity of sulphuretted hydrogen formed corresponded to 0.0281 gr. sulphur or 70 % of the quantity present in 2 grammes of P_1. 1.945 grammes of carbon were recovered, which contained 0.47 % sulphur. Hence 23 % of the sulphur present in P_1 has remained behind in the carbon that is left.

In a third experiment 2 grammes of P_1 were heated for some days in a hydrogen current. The temperature was between 500—700° for 9 hours, and between 700—800° for 11 hours. Throughout the experiment sulphuretted hydrogen was regularly generated. The remaining carbon still contained 0.29 % sulphur; it is, therefore, possible to convert practically all the sulphur from P_1 to sulphuretted hydrogen by prolonged heating in hydrogen at 500—800°.

To verify whether really a particular action of hydrogen should be assumed here, 1 gramme of P_1 was heated in a current of pure dry nitrogen at 900—990° for 8 hours; this appeared to have reduced the sulphur content only little. The percentage of sulphur found

in the remaining product was 1,75 %. It follows from this last experiment that the formation of sulphuretted hydrogen does not take place in this manner, that primarily sulphur vapour is split off, which reacts with the hydrogen. For if the sulphur combined with the carbon had an appreciable vapour tension at 900°, the generated sulphur vapour would be carried along by the nitrogen current, and then it would be possible to remove nearly all the sulphur from the carbon by heating in a nitrogen current, which is not the case. In the action of hydrogen on P_1 we have, therefore, to do with a specific action of the hydrogen, hence with a chemical reaction.

§ 4. I then examined more closely the conditions under which, and the temperatures at which this fixation of sulphur by amorphous carbon takes place. These experiments have not yet led to a satisfactory insight into the proposed problem, and should, therefore, be considered as provisional. A series of experiments was arranged as follows:

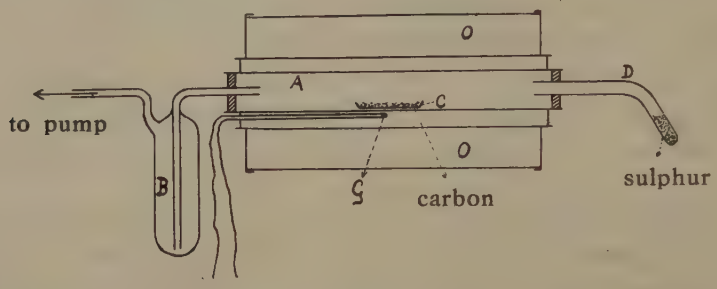
1.5—2 grammes of a mixture of 75 % amorphous carbon K_1 and 25 % sulphur was put in a porcelain boat, this boat was placed in a porcelain tube, which was in an oven heated beforehand at a definite temperature. The oven was then kept at this temperature for an hour, pure nitrogen flowing through the tube during this time; then the tube was cooled in a nitrogen current. This experiment was made at different temperatures (all above the boiling-point of sulphur), viz. at 500—510°, at 610—590°, 670—710°, 900—940°.

Most sulphur distilled from the boat for the greater part already at the beginning of the experiment. In the experiment at 500—510° no H_2S-formation was observed; it was, however, observed in the experiments at 610—590° and at the higher temperatures.

In all these cases the carbon recovered after the experiment had fixed no sulphur or very little. Compare with this the preparation of P_1 (§ 2), in which a larger quantity of mixture of carbon + sulphur was placed in an oven heated at 510°, which temperature was slowly raised to 975°; in this latter case the contents of the porcelain tube will have assumed the temperature of 510° less rapidly, and the sulphur could, therefore, be fixed by the carbon, before all the sulphur had been distilled out from the mixture.

The temperature at which the fixation took place in this experiment, cannot be ascertained. In the series of experiments mentioned in § 4 the small quantity of substance quickly assumed the temperature of the heated tube, the sulphur evaporated almost immediately after the boat had been pushed into the oven, the time during

which the sulphur vapour was in contact with the heated carbon was evidently too short for the fixation of sulphur by carbon to take place. For this reason some experiments were made in which the sulphur vapour was longer in contact with the carbon heated at a definite temperature.



In the middle of a porcelain tube *A* there is a boat *C* with about 2 grammes of amorphous carbon. The junction *G* of a thermo-element is on the outside of the porcelain tube at a level with the boat. *O* is an electrical oven. On one side a tube of sparingly fusible glass *D* was fastened air-tight in the porcelain tube. The bent part of this tube was partly filled with sulphur. On the other side the tube was in connection with a receiver *B*, which could be cooled during the experiment, and which was also connected with the vacuum pump. First the whole apparatus was evacuated to ± 2mm., and then the heating was started. When the desired temperature had been reached, it was kept constant and — while evacuation was continued — the tube *D* was heated, so that the sulphur began to distill through the porcelain tube. In this way sulphur vapour flowed over carbon that was heated at a definite temperature. After the sulphur had been distilled over, the apparatus was left to cool in vacuum.

I. Carbon heated at 550°. ± 6 grammes of sulphur distilled over during 30 min. Original weight of carbon found back: it contained 0.91 % sulphur.

II. Carbon heated at 885°. ± 4 grammes of sulphur distilled over during 15 min., the product obtained contained 1.53% sulphur. In these experiments most of the distilled sulphur condensed in the colder part of the tube, a little in the receiver which was cooled in carbonic acid and alcohol. Formation of carbon disulphide (CS_2) was not observed. If any was formed which did not condense at —80° and 2 mm., the quantity must have been very small, because the greater part of the carbon was found back.

It accordingly appears from these experiments that sulphur is

fixed by carbon both at 550° and at 885°. The influence of time and temperature will be decided by further experiments.

5. The experiments described may be interpreted in different ways. The sulphur may have been absorbed by the carbon or fixed by chemical forces. Let us first consider the former case more closely. Amorphous carbon is an exceedingly efficient absorbent for various gases[1]), why should not sulphur vapour be absorbed? By absorption is understood a reversible phenomenon; the absorbed gas is condensed on the absorbent, and has remained unchanged in its chemical properties.

The fact that the carbon does not lose its sulphur-content at 1000° and 1 mm. pressure is not conclusive against the assumption of absorption. For if the absorption-isotherm has an asymptote in the axis of coordinates, it is possible that the last rests of absorbed substance (in this case 2 % sulphur) are practically not to be removed. The influence of the temperature and the time during which the sulphur vapour is in contact with the carbon, on the quantity of sulphur which is fixed, will have to be studied more closely to render it possible to draw a definitive conclusion in this respect. The behaviour of the fixed sulphur towards hydrogen, however, seems to me an argument in favour of the sulphur being fixed to the carbon by chemical forces. By the action of hydrogen on the sulphur-carbon complex sulphuretted hydrogen is formed. In this reaction it must be assumed that the hydrogen reacts with the solid phase. For the sulphur vapour tension of the sulphur-carbon complex is still very small at 900°, otherwise finally all the sulphur would be expelled in gaseous form by heating in a nitrogen current at that temperature. For the same reason the sulphur cannot be assumed to be in the carbon in a solid solution having a certain vapour-tension. In this case, too, it would be incomprehensible why the sulphur could be expelled by hydrogen, and not by nitrogen.

Also the fact that by treatment of the product P_1 with bromine water only a small part of the sulphur is oxidized, is difficult to reconcile with the idea of absorption.

I assume for the present that the sulphur is bound to the carbon by chemical forces, and propose to designate the carbon-sulphur compound formed in this way for the present by the name of carbon sulphide. Nothing can be concluded with regard to the composition

[1]) This holds at least for charcoal, animal charcoal and such substances. Whether also pure amorphous carbon is a good absorbent, does not seem to have been decided as yet.

of this carbon sulphide from the experiments described. A priori it does not even seem established that there is question here of one or more compounds of constant composition. Besides it seems possible that a fixation by chemical forces has to do something with surface phenomena. In LANGMUIR's experiments [1]) on the fixation of oxygen by heated filament of carbon or tungsten one has to do with a very thin layer of oxygen, which is retained at the surface of the carbon or tungsten filament by chemical forces.

LOWRY and HULETT [2]) have shown that amorphous carbon (in their case not entirely pure) can fix oxygen at 25° in another way than by absorption. Even at 180° this oxygen could not be pumped off from the carbon, at higher temperatures the oxygen split off as CO and CO_2. Some years ago, RHEAD and WHEELER [3]) have shown that oxygen can be fixed by amorphous carbon at temperatures between 250° and 500°, and that on heating of this carbon-oxygen complex CO and CO_2 is formed. In these researches it has been established beyond doubt that the fixation of oxygen to the carbon takes place by chemical forces. The quantity of oxygen fixed in this way in LOWRY and HULETT's experiments, was 1.7—3.75 of the weight of the carbon, a quantity which is, therefore, of the same order of magnitude as in the sulphur-carbon complexes. An analogy between these solid carbon-oxides and the carbon-sulphides described in this communication is undeniable. Whether these carbon sulphides also on still higher and more prolonged heating split off the sulphur as volatile carbon-sulphide compounds, would have to be decided by further experiments.

It is possible that the sulphur atoms are bound by rest-valencies of the carbon atoms which have remained unsaturated after the combination of the carbon atoms to amorphous carbon. This rest-affinity will possibly not be the same for different preparations of amorphous carbon, but depend on the way in which the amorphous carbon has been obtained.

Led by this idea I have made similar carbon sulphides starting from sulphur and from sugar-carbon, which had been purified by being heated successively in a current of chlorine, a current of hydrogen and in vacuum. In this way similar, but quantitatively different, results were obtained. By heating of charcoal with sulphur first a large quantity of sulphuretted hydrogen was developed, and finally a carbonsulphide resulted containing 3,5 %

[1]) Journ. Amer. Chem. Soc. **37**, 1154 (1915) and **38**, 2271 (1916).

[2]) Journ Amer. Chem, Soc. **42**. 1408 (1920).

[3]) Journ. Chem. Soc. **101**, 831 (1912), **103**, 461.

sulphur; this sulphur-content could be reduced to 2,7 % by heating in vacuum, but remained then constant. Its behaviour towards oxidizers was perfectly analogous to that of the carbon sulphide described in § 2. These experiments, which are still to be confirmed by a new series will be described later.

§ 6. These carbon sulphides obtained synthetically present a close resemblance with coal-coke, for so far as the sulphur-content is considered. The sulphur fixed to carbon in coal-coke is very resistent to oxidizers and cannot be expelled by heating.

In a recent research POWELL [1]) has shown that the sulphur content of coke can be considerably reduced by leading hydrogen over the coke at 500—1000°, which caused the sulphur fixed to carbon to be transformed into sulphuretted hydrogen for a large part. His statement [2]) that sulphur-free coke can combine with sulphur when heated in a mixture of hydrogen and sulphuretted hydrogen, and that there is equilibrium between the carbon-sulphide and the sulphur vapour, cannot be judged until the experiments have been further described.

In conclusion I will mention the product that STOCK and PRAETORIUS [3]) obtained in their research on the carbonsubsulphide (C_2S_2). This carbonsubsulphide polymerizes to a black mass of unknown molecular weight. On heating this carbon-like substance to dark redness CS_2 and C_3S_2 escaped; but there remained a carbon-like mass that contained 39 % sulphur. A closer examination of such a product and its behaviour at prolonged heating will be of interest.

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[1]) Journ. of Ind. and Engin. Chem. **12**, 1077 (1920).
[2]) Ibid. **13**, 34 (1921).
[3]) Ber. **45**, 3569 (1912).

Physics. — *"An Extension of the Theory of* BABINET'S *Compensator."* By C. A. REESER and Prof. R. SISSINGH. (Communicated by Prof. H. A. LORENTZ.)

(Communicated at the meeting of June 25, 1921).

1. For an examination of elliptically polarized light, BABINET'S compensator must satisfy the condition that the principal planes of the two wedges are at right angles to each other. Besides, if this elliptically polarized light arises from reflection, one of the principal planes must coincide with the plane of incidence of the mirror. For this purpose one of us has used the dark line in the field of polarisation of the nicol, which was first observed as band by LANDOLT, and afterwards studied by LIPPICH [1]). In an experimental investigation on the true optical constants of mercury, carried out by REESER, it appeared, however, that phenomena, which had not been observed before in the compensator, can successfully be used in the adjustment of the compensator, which leads to greater accuracy [2]). The phenomena in question have first been experimentally studied, and then theoretically confirmed.

2. The above mentioned phenomena are obtained by means of a cylindrical beam of rays, which is not perfectly achromatic. For this purpose a spectrum is thrown on the slit of a collimator, and the beams of light, issuing from it, are made to traverse a compensator placed on a goniometer between two nicols. Let the compensator at first contain only one wedge. The front plane of this wedge is at right angles to the beam of rays.

The analyzer is placed behind the wedge and also a telescope, which is adjusted for parallel rays. The change of direction, which the rays of light undergo in their passage through the polarizer, has been for the greater part neutralized by the aid of two glass wedges [2]). The polarisation-planes of the two nicols are about normal to each other and the wedge is adjusted so, that the illumination of the

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[1]) LIPPICH, Wiener Sitz, Ber., 85, 1882; 91, 1885. SISSINGH, Proefschrift, Leiden, 1885; Arch. Néerl., 20. 1886.

[2]) C. A. REESER. Proefschrift, Amsterdam. 1921.

field, i.e. of the image of the slit in the telescope, is a minimum. One of the planes of polarisation of the nicols is then about parallel to and the other normal to the principal plane of the wedge.

3. The collimator-slit takes a part out of the spectrum, the wave-length of which varies a little in a direction normal to the slit. Thus different cylindrical beams of light issue from the collimator, which differ somewhat in wave-length, and the axes of which form small angles with each other. The image of the slit in the telescope is, therefore, part of a spectrum. In this image there are seen some black lines, parallel to the longitudinal direction of the slit. In our case the number, depending on the width of the collimator-slit, was three. By experiment the following properties of these lines were found:

a. The lines arise from the extinction of definite colours. They become sharper as the purity of the spectrum increases and more numerous with greater width of the slit.

b. The wedge-form of the quartz wedge is without influence, as a slit before the quartz wedge does not change the phenomenon.

c. For one analyzer-position there are found two polarizer-positions, for which a system of black lines is observed. The lines of one system lie between those of the other. The angle between the two polarizer-positions is smaller as the principal plane of the wedge coincides more with the plane of polarisation of the polarizer or is normal to it. The middle-plane between the two polarizer-positions forms an angle of 0° or 90° with the principal plane of the wedge. One of the two polarizer-positions is only normal to the analyzer, independent of the position of the principal plane of the wedge.

d. When the plane of polarisation of the analyzer makes an angle of 0° or 90° with the principal plane of the wedge, the dark line in the polarisation-field of the two Nicols (§ 1) is visible in the slit-image. Supposing that this is about at right angles to the edges of the slit, the two systems of black lines are visible at the same time, one above, the other under the nicol line. This forms so to say, a transition between the two systems. The lines seem thickened there.

e. On rotation of the polarizer from one to the other of the two positions, mentioned in *c*, the lines first become fainter, then they rapidly move through the field to those of the second system, in which the latter become very black.

c suggests immediately a mode of procedure to place the wedge so, that the principal plane of the wedge coincides with the polarisation plane of one of the nicols, and is normal to that of the other. This position is obtained with successive approximations.

A simple consideration shows immediately, that one system of lines is formed by the rays, in which the phase difference in the wedge is $2k\pi$ ($k =$ integer), the other by those with a phase difference $(2k+1)\pi$. The properties mentioned in c and d follow immediately from this.

4. When all the rays of the broad incident beam are supposed to pass over an equally long path in the wedge, an idea of the state of polarisation behind the wedge is obtained in the following way. Let the angle between polarizer and principal plane of the quartz plate be α, between analyzer and principal plane ψ. From the wedge issues elliptically polarized light. Let one of the axes form an angle θ with the principal plane. Let $\cos \mu t$ be the light vector in the incident light and $A \cos(\mu t - \chi)$, $B \sin(\mu t - \chi)$ that according to the axes of the elliptically polarized light, issuing from the wedge. Let δ denote the difference of phase between the ordinary and the extraordinary ray in the wedge. Then:

$$\cos\alpha = A\cos\chi\cos\theta + B\sin\chi\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\cos\delta\sin\alpha = A\cos\chi\sin\theta - B\sin\chi\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$0 = A\sin\chi\cos\theta - B\cos\chi\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\sin\delta\sin\alpha = A\sin\chi\sin\theta + B\cos\chi\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From this set of equations follows:

I. $A^2 + B^2 = 1$, II. $\cos 2\alpha = (A^2 - B^2)\cos 2\theta$, II$_a$. $AB = \sin 2\alpha \sin\delta$,

III. $tg\, 2\theta = tg\, 2\alpha \cos\delta$, IV. $\sin 2\chi = tg\,\alpha \sin\delta \sin 2\theta$.

I is a consequence of the supposition, that nowhere light is absorbed. The light issuing from the wedge is polarized rectilinearly, when $A = 0$. Hence there appear black lines, if:

$A = 0$, therefore $B = 1$, for $\alpha = \frac{1}{2}\pi + \theta$ $\delta = 2k\pi$

or $\alpha = \frac{1}{2}\pi - \theta$ $\delta = (2k+1)\pi$.

A special solution is $A = 0$, $\alpha = \frac{1}{2}\pi$, δ indefinite.

In this way the properties in a—d have been found back.

5. From the analyzer issue two rays of light with amplitudes $\cos\alpha\cos\psi$ and $\sin\alpha\sin\psi$, with difference of phase δ, so that the intensity is

$$I = \tfrac{1}{2}(1 + \cos 2\alpha\cos 2\psi + \sin 2\alpha\sin 2\psi\cos\delta).$$

This expression shows again, that there are two systems of dark lines for $\delta = 2k\pi$ and $\delta = (2k+1)\pi$. The transition of one system to the other is such, that one system of lines becomes brighter, the other darker. A shifting, as described in e, does not take place

however. In order to explain this, the wedge-form of the quartz plate should be taken into account.

6. The distance of the lines in the field of the compensator with two wedges, placed in opposite direction, having equal angles and their principal planes perpendicular to each other, shows that the difference of phase in the wedge varies from one rim of the beam to the other by π, the width of the incident beam being about 8 mm. The angle of the wedges is 15′. Call this ε, then for two rays with mutual distance x, the change of the difference in the paths in the wedge becomes $d_1 - d_0 = \varepsilon x$ and the change of the phase-differences is determined by $\delta_1 = \delta_0(1 + \varepsilon_1 x)$, in which $\varepsilon_1 = \varepsilon : d_0$. In this d_0 is the path, that one of the rays passes over in the wedge. Let α be $= \frac{\pi}{2} + \Delta$. Let ε_1 and Δ be taken as infinitely small. Let further the amplitude of the beam of light in the focal plane of the object lens behind the wedge be called 1, when the incident beam is 1 mm. wide. Then from the equation in (4) follows, with neglect of quantities of the 2nd order:

$$A = -\Delta \sin \delta_0(1 + \varepsilon_1 x); \quad B = 1 - \tfrac{1}{2}\Delta^2 \sin^2 \delta_1(1 + \varepsilon_1 x);$$

$$\theta = \Delta \cos \delta_0(1 + \varepsilon_1 x); \quad \chi = -\tfrac{1}{2}\pi + \delta_0(1 + \varepsilon_1 x).$$

From the analyzer issue beams of light with amplitudes $A\cos(\psi - \theta)dx$ and $B\sin(\psi - \theta)dx$ and phases χ and $\frac{1}{2}\pi + \chi$. In order to obtain the value of the light-vector in the focal plane, these amplitudes must be composed, and then we have to integrate over the width x of the beam. We can, however, immediately write down the components of the total amplitude in two directions at right angles to each other, viz.

$$X = \int_0^x [A \cos(\psi - \theta) \cos\chi - B \sin(\psi - \theta) \sin\chi]\, dx$$

$$Y = \int_0^x [A \cos(\psi - \theta) \sin\chi + B \sin(\psi - \theta) \cos\chi]\, dx.$$

By substitution of the values A, B, χ, and θ indicated here, and neglect of quantities of the 2nd order, we get for the intensity of the light:

$$I = x^2 \left[\Delta^2 + \Delta_0^2 - 2\Delta\Delta_0 \cos\delta_0 \left(1 + \frac{\varepsilon_1 x}{2}\right)\right].$$

In this:

$$\Delta_0 = \frac{2\psi}{\varepsilon_1 \delta_0 x} \sin \frac{\varepsilon_1 \delta_0 x}{2}.$$

There appear black lines, if $X = Y = 0$, so that:

$$\delta_0 \left(1 + \frac{\varepsilon_1 x}{2}\right) = 2k\pi, \quad \Delta = \Delta_0$$

or

$$\delta_0 \left(1 + \frac{\varepsilon_1 x}{2}\right) = (2k_1 - 1)\pi, \quad \Delta = -\Delta_0.$$

It thus appears, that $\frac{1}{2}\pi + \Delta_0$ and $\frac{1}{2}\pi - \Delta_0$ are the azimuths of the two polarizer-positions, for which black lines appear.

The wave-lengths, for which minima appear at given polarizer-positions, follow from $dI : d\delta_0 = 0$. This yields $\Delta = C\Delta_0 : (C\cos\varphi + \varphi \sin\varphi)$, hence:

$$C = 1 - \frac{\varepsilon_1 \delta_0 x}{2} \cot \frac{\varepsilon_1 \delta_0 x}{2} \quad ; \quad \varphi = \delta_0 \left(1 + \frac{\varepsilon_1 x}{2}\right).$$

$\varepsilon_1 \delta_0 x$ varying little with the wave-length, C is independent of it. The difference of phase in the quartz wedge for a thickness d_1 is $d_1(n_g - n_b) : \lambda$, in which $n_g - n_b = 0{,}009$, $\lambda = 6 \times 10^{-4}$. For $d_1 = 3$ mm., which value is certainly not exceeded, the difference of phase is $45 \times 2\pi$. Further $\varepsilon = 15'$, or in radians $1 : 240$, $\varepsilon_1 = \varepsilon : d_0$ about $1 : 720$, hence for $x = 8$ mm., $\varepsilon_1 \delta_0 x$ smaller than π. Thus C is always smaller than 1.

For $\Delta = \Delta_0$ the minima appear at $\varphi = 2k\pi$, and I then being $= 0$, they are perfectly black. It appears from the foregoing, that k is about 45.

When we start from perfectly black bands, $\Delta = \Delta_0$, $\varphi = 2k\pi$, $I = 0$, and let φ increase by $\Delta\varphi$, so that $C + \varphi\, tg\, \varphi = 0$, then $\Delta\varphi = -C : 2k\pi$, and Δ has risen to a very great value, likewise I, which depends on Δ^2. It appears from this, that the dark bands get very much fainter on increase of the azimuth $\frac{1}{2}\pi + \Delta_0$ of the polarizer, but are hardly displaced at all. When now φ is increased by a very small amount $\Delta\varphi = C : 2k\pi$, so that $\varphi\, tg\, \varphi = C$, then Δ is $= \frac{1}{2}\Delta_0$, $I = \frac{1}{4}\Delta_0^2$. Accordingly the dark bands are very little displaced, when the azimuth of the polarizer decreases from $\frac{1}{2}\pi + \Delta_0$ to $\frac{1}{2}\pi + \frac{1}{2}\Delta_0$. If the bands have to shift to places for which $\varphi = (2k + \frac{1}{2})\pi$, Δ must decrease to $C\Delta_0 : (2k + \frac{1}{2})\pi$, or about $\Delta_0 : 300$, for $C = 1$, $k = 45$. In order, therefore, to displace the minima over a fourth of their distance, Δ must decrease to a very small fraction of Δ_0. The bands, which at first seem to remain in their places, when the polarizer is rotated, afterwards move through the field with great rapidity. Theory thus explains the phenomena given in 3*e*. We have further examined, what happens to the bands at the places for which $\varphi = (2k + 1)\pi$ at $\Delta = -\Delta_0$.

The results, however, are not recorded, because the phenomena take place at such small values of Δ, that they are difficult to observe experimentally.

7. Similar results, as have been described in § 3 for one wedge, are observed with both compensator-wedges.

a. In the slit-image black bands can appear for suitable nicol-positions; only one position of the polarizer, however, corresponds to a definite position of the analyzer. Compare 3*c*.

b. These positions of the nicols are confined to a region, which becomes smaller, as the angle between the principal planes of the wedges departs less from 90°.

From this follows a new criterion for the adjustment of the true position of the wedges (angle 90° between the principal planes), viz. that the black bands disappear on the slightest displacement of the polarizer.

We may refer for the theoretical explanation to a thesis for the doctorate by C. A. Reeser, Amsterdam, 1921. In this the phenomena corresponding to 3*e*, which Reeser termed fading-phenomena, i.e. in which the dark bands are fading away, were left out of consideration as exceedingly complicated. The theory is therefore restricted to the occurrence of perfectly black bands.

Physics. — "*The Optical Investigation of Surface Layers on Mercury and a More Refined Method of Observation with* BABINET's *Compensator.*" By C. A. REESER and Prof. R. SISSINGH. (Communicated by Prof. H. A. LORENTZ.)

(Communicated at the meeting of June 25, 1921).

1. This research was made with the same apparatus as HAAK used [1]). It will, therefore, only be mentioned here, what modifications have been made in it. In order to obtain a greater light-intensity and to have greater certainty, that the wave-length of the light used was not modified during the observations, the monochromator, placed before the goniometer, was omitted. The light, which must be monochromatic as much as possible, was now obtained by means of colour-filters or a mercury lamp of HERAEUS (3,5 A, 35 V). As source of light we finally returned to the self-regulating arc-lamp of 18 Ampères [2]). The collimator-axis was in the right direction of incidence, so that the mirror behind the collimator could be omitted.

2. As is known, the nicols with oblique end-planes give, besides a displacement, also a change of direction to the beam of rays in consequence of errors in the construction. It is about $^1/_3{}^\circ$ in the nicols used. This gives rise not only to a change in the direction of incidence, but, in using a monochromator, also to a shifting of the compensator line. The collimator slit takes out of the spectrum a small portion, the wave-length of which varies in the direction of the width of the slit. In consequence of the deviation by the polariser not always the same part nor light with the same wave-length falls between the threads of the compensator. In the most unfavourable case a shifting of the line took place in the compensator in consequence of this, corresponding to a phase-difference of $0{,}02 \times 2\pi$.

In order to eliminate this difficulty Mr. REESER applied the following expedient. Two glass wedges with refractive angles of 1° were placed before the polariser. The angle between the planes, perpen-

[1]) J. J. HAAK. Proefschrift Amsterdam, 1918; These Proc., Vol. XXI., No. 5, p. 678 (1918).

[2]) Compare for further particulars. C. A. REESER Proefschrift. Amsterdam 1921.

dicular to their refractive edges, could be given any value. When by this means the same deviation is given to the ray of light thrown on the polariser, as is caused by the polariser itself but in opposite direction, the deviation is neutralized.

3. For the observation of the lines in the compensator, a slit is placed before it and the threads, to shut off the strong light, that passes on the side of the threads through the compensator. A slit of 1 mm. is sufficient. A narrower slit gives troublesome diffraction phenomena. A shifting of this slit before the compensator gives a change in the position of the compensator-lines. Therefore the slit must always be symmetrical with regard to the threads. As, however, slit and threads are not in the same plane, this symmetrical position is disturbed, when the rays run no longer in the direction of the axis of the telescope behind the analyzer. Thus before every determination of the position of the compensator-lines the telescope must be carefully placed in the direction of the reflected rays. If this precaution is observed, the positions of the compensator lines in the different polariser positions differ at most so much as corresponds to a phase-difference of $0{,}0013 \times 2\pi$, the limit of observability being $0.0006 \times 2\pi$.

4. Finally Mr. Reeser has still applied the following expedient to make the method of observation more precise. With a telescope behind the compensator, focussed for parallel rays, we see besides the direct slit image, two more side images, formed through internal reflection in each of the wedges. By focussing on the compensator lines, hence on the threads, the side images coincide, however, with the central, direct one. It is self-evident that this is not conducive to an accurate measurement of phase-difference and the ratio of the components of the examined elliptically polarized light. When the rays of these side-images are shut off by a screen with a slit in the focal plane of the object-lens of the telescope, the compensator line becomes perfectly black, narrower and more sharply defined. The accuracy of only one determination of the position of the line corresponds to $0.0006^5 \times 2\pi$; the accuracy of the restored azimuth, which was before about 20′ for 64 adjustments, now amounts already to 5′—9′ for 16 adjustments. We have found this increased accuracy to be absolutely necessary for the investigation of the surface-layers.

5. In order to prevent vibrations of the mercury surface, the mercury vessel is mounted on a small adjustable stand, free from the goniometer. The displacements of the nicols and the compensator can then take place without danger of vibrations of the liquid surface.

When once the goniometer axis has been put at right angles to the normal of the mercury surface, this is the case for each new surface [1]). The two following criteria, derived by DRUDE, refer to the purity of a mercury surface: *a.* surface layers increase the reading of the compensator. *b.* increase the restored azimuth at not too small angles of incidence [2]). To this the following criterion, found experimentally, may be added. For a pure mercury surface, which as such gives the lowest value, both for the phase-difference and for the restored azimuth, the reading of the compensator in the air may vary only slowly. If the mercury is not perfectly pure, a clearly perceptible rise of the compensator-reading, corresponding to a phase-difference of $2\pi : 150$, is observed soon after the formation of the mercury surface, even in a space of greatly rarefied air. That this rise does not take place through formation of an adhering layer of air, appears also from this, that it proceeds incomparably much more quickly than is the case with that in consequence of anairlayer, which forms slowly, according to a relation given and tested by HAAK [3]). The only conceivable explanation is the coming to the surface of impurities. No sufficient improvement is attained by rubbing off the mercury-surface with cotton wool, so that the upper layer is removed. Addition of clean mercury influences the surface-layer only to a very small degree. This experience leads to the conclusion that RÖNTGEN's pouring-out method and that of the communicating vessels of DRUDE cannot give a sufficiently pure mercury surface. Better is the method that WERNICKE applied to crystals and glass, in which a solution of collodion in ether is poured over the surface and the film, which is formed after half an hour, is detached from the side of the mercury vessel, and slipped off over the mercury. This procedure, however, also fails several times, and is often less successful on repeated use of the same glass and mercury. The best results were obtained by conveying double distilled mercury after filtering through a paper funnel into a glass vessel with a fine, drawn-out point. This vessel fits with a ground rim into a cylinder glass, which is also filled with mercury through the fine point. The vessel itself is closed by means of a ground-in stopper. From the vessel the mercury is run into a glass dish with flat bottom and small depth, which had been carefully cleaned previously. The dish is kept between filter paper and never touched. Alcohol is never used for the cleaning, for fear of impurities and oxidation of the mercury.

[1]) C.f. for the centering HAAK, loc. cit.

[2]) DRUDE, Wied. Ann., 39, 492. 1892.

[3]) HAAK, Thesis, p. 32.

6. The observations have been carried out with four light-filters. The wave-lengths of the maxima of the transmitted narrow spectral regions are successively 669 $\mu\mu$; 637 $\mu\mu$; 558 $\mu\mu$ and 482 $\mu\mu$. Besides, the yellow, green, and blue mercury lines have been used by the aid of the quartz mercury lamp. Its wave-lengths are 578 $\mu\mu$, 546 $\mu\mu$ and 436 $\mu\mu$. For the way in which the angle of incidence and the lowest compensator reading is obtained, compare the thesis for the doctorate of Mr. REESER. Let us call the lowest compensator reading for a mercury mirror without surface layer c', the observed reading c, ψ' and ψ the corresponding values of the restored azimuth, then $(\psi'-\psi):(c'-c)$ can be calculated with the formulae in § 8, so that ψ' and c' can be derived from the observations.

7. First the optical constants, principal angle of incidence I and principal azimuth H, also n_0 and k_0, index of refraction, and coëfficient of absorption in a direction normal to the boundary plane have been determined for a pure mercury surface. The phase-difference φ and the restored azimuth ψ is determined with each of the colours for five or six angles of incidence. From this $n\cos\alpha$ and k are calculated for every angle of incidence, in which n and k are index of refraction and coefficient of absorption corresponding to the angle of incidence, and α represents the angle of refraction. The mean is calculated from the different values of $n\cos\alpha$ and k for the same angle of incidence, and thus I and H according to the relations

$$tg\,2H = \frac{k}{n\cos\alpha} \quad , \quad tg\,I = \frac{n\cos\alpha}{\cos 2H \sin I}$$

The second equation gives I through successive approximations [1]). Here follow the values of I, H, n_0 and k_0 for three colours, viz. the yellow, green, and blue mercury lines.

yellow mercury line	$I=79°28'$	$H=35°50'$	$n_0=1{,}693$	$k_0=4{,}934$	$\lambda=578\,\mu\mu$
green ,,	78°56'	36°16'	1,538	4,696	$=546$,,
blue ,,	76°21'	38°2'	0,995	3,754	$=436$,,

The accuracy of I is for the different colours from 3′ to 8′, for H from 6′ to 8′, for n_0 from 1%—2%, for k_0 from ½% to 1%. The values here obtained are compared with those of earlier investigators, and it is shown that in most of them the mercury had a surface layer [1]).

[1]) See for further information the thesis for the doctorate of C. A. REESER, Amsterdam 1921.

8. The surface layers of smoke, air, and oil upon mercury have been examined by means of the more delicate method of observation (see 2, 3, and 4). For a surface layer which is so thin, that the index of refraction may be considered as constant, the following equations may be derived from DRUDE's theoretical considerations [1]):

$$\frac{\psi_2 - \psi_1}{c_2 - c_1} = \frac{27}{x} \sin 2\psi \frac{a'}{\cos^2 i - a} (1 - n^2 \cos^2 i)$$

$$c_2 - c_1 = \frac{400\, x}{\lambda} \cos i \sin^2 i \frac{\cos^2 i - a}{(\cos^2 i - a)^2 + a'^2} \left(1 - \frac{1}{n^2}\right) (l_2 - l_1).$$

In this ψ' and ψ are the restored azimuths for a clean mercury surface and one with a surface layer, c' and c the corresponding compensator readings, x the compensator displacement for a phase-difference $\frac{1}{4}$, i the angle of incidence, n the index of refraction of the surface layer, $l_2 - l_1$ its thickness, a and a' are determined by:

$$a = \frac{\cos 4H}{\sin^2 I \operatorname{tg}^2 I} \quad , \quad a' = \frac{\sin 4H}{\sin^2 I \operatorname{tg}^2 I}.$$

Seven surface-layers of smoke have been examined. Two have been obtained by blowing more smoke on to an already existing smoke layer. Smoke is blown into a pipette, provided with a bulb, which ends above the mercury in a fine point. The determinations of phase-difference and restored azimuth with compensator and polarizer (the azimuth of the analyzer is always 45°) are carried out successively on pure and smoked mercury under circumstances, which are the same down to the minutest particulars. The values of l obtained for the different colours with the same layer do not diverge too much. $10^3 l : \lambda$ varied from 3,15 for the thinnest to 34,4 for the thickest layer. The values of n range for every colour with increasing thickness of the layer from about $4 \rightarrow 5$ to $2 \rightarrow 2.5$. The graphical representations point to the existence of a maximum for a definite value of l. For all layers l remains below the limiting value, about $0{,}03\,\lambda$, for which the formula may still be applied.

Two layers of air have been examined. In these circumstances the mercury was protected by a glass plate, so that particles of dust and fat are excluded. The thicknesses of the layers were 2.2×10^{-6} and 3.4×10^{-6} mm., n ranges for the different colours from $2.6 \rightarrow 4.4$ to $3.4 \rightarrow 5.5$.

Besides bone-oil has been communicated to the mercury-surface and the spreading of the oil has been examined. The results have been

[1]) DRUDE, Wied. Ann., 39. 1890, 488.

compared with FISCHER's observations on mercury [1]) and DEVAUX' observations on water [2]). By means of the compensator it can be shown, what FISCHER suspected, that a small quantity of oil on mercury immediately spreads on the surface in a very thin layer. The thickness of the very thin layer is 1—2 $\mu\mu$. FISCHER calls it the "vorauseilende Schicht".

The thickness of the thinnest layers of smoke, air, and oil are 1.6 $\mu\mu$; 2.05 $\mu\mu$, and 1.07 $\mu\mu$. The values obtained for the index of refraction in all layers always point to a very great density of the layer at the mercury surface.

Whereas with capillary phenomena, as the cessation of the movements of the camphor particles on water, the thinnest layer that can be observed, is 1 $\mu\mu$, it appears that the presence of a layer of 0.3 $\mu\mu$ can very well be verified with the compensator. The optical method is, therefore, more minute, which was also found by RAYLEIGH. In conclusion it should be pointed out that in the calculation of the thickness of the layers from the number of milligrammes, the greater density of the layer at the surface is, as a rule, left out of account.

All the observations have been made by the former of us.

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[1]) FISCHER. Wied. Ann., 68, 436. 1899.

[2]) DEVAUX. Journ. de phys. (5), II, 699, 1912.

Chemistry. — "*The Use of the* ZEISS *Waterinterferometer* (RAYLEIGH-LÖWE) *for the Analysis of Non-Aqueous Solutions*". By Prof. ERNST COHEN and H. R. BRUINS.

(Communicated at the meeting of June 25, 1921).

1. Among the methods for a quantitative determination of the concentration of solutions, the optical methods excel the others in many cases in accuracy and rapidity of execution. With very careful regulation of the temperature (constant down to 0°.01) it is possible to determine with the most practical measuring instrument, the refractometer, indices of refraction accurate to 1 or 2 units of the fifth place of decimals, which about corresponds in aqueous salt solutions with an error in the determination of the concentration of 0.02 %.

In some cases, e.g., for the analysis of exceedingly diluted solutions, this accuracy is, however, not sufficient. An instrument, which is eminently fit for such determinations, is the waterinterferometer according to RAYLEIGH-LÖWE, put on the market by the firm ZEISS. It enables us to measure the index of refraction of a solution down to 2 units in the 7th place of decimals, corresponding to an error in the determination of the concentration of at most 0.0002 %. This interferometer is, however, as its name indicates, constructed for the use of water as solvent, and all the investigations which have been executed by the aid of it up to now, concerned aqueous, or very diluted alcohol, solutions.

In connection with an investigation, of which we hope soon to give further details, it was necessary to carry out accurate analyses of exceedingly diluted solutions in organic liquids. It then appeared that if the interferometer is to be used also in this case, a number of precautions must be observed, which will be set forth more at length in what follows.

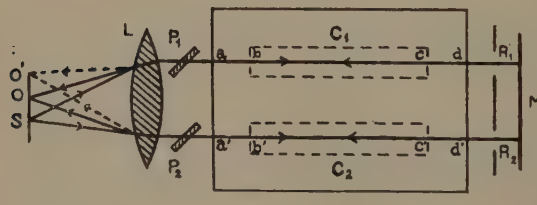

Fig. 1.

We will, however, first give a short description of the interferometer with the aid of figures 1 and 2 [1]).

2. Fig. 1 gives a schematic representation of the instrument. The rays furnished by a source of light S, made parallel by lens L, pass through the identical vessels C_1 and C_2 filled with the same liquid, and then through a screen provided with two slits R_1 and R_2, are reflected as two separate beams by mirror M, and after having been united again by lens L, they form an interference image in O. When white light is used this image consists of a central bright band bounded by two dark ones; the bright bands following on them on either side have coloured edges. When now C_1 is filled with a solution which has a greater index of refraction than the pure solvent in C_2, this interference image is displaced in consequence of the lengthening of the optical path, e.g. to O'. It can be brought back to the zero position O by turning the compensator plate P_1 (P_2 is immovable), through which the optical path is again artificially shortened. The angle, over which P_1 has been rotated, is a measure for the difference of index, hence indirectly for the concentration of the solution in C_1.

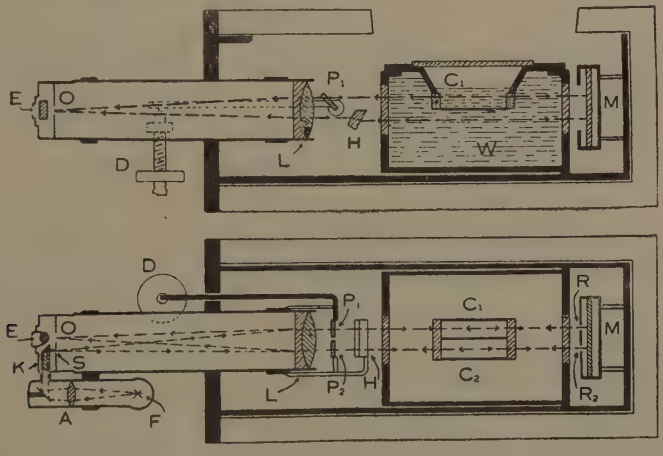

Fig. 2.

Fig. 2 represents a horizontal and a vertical cross-section of the interferometer more in details. The white light of an Osram lamp F is concentrated by lens A and prism K, on the very narrow slit S, which forms the secondary source of light. The passage of the

[1]) A fuller description is found in the following places:

F. Löwe, Zeitschr. für Instrumentenkunde **1910**, 321; F. Löwe, Physikalische Zeitschr. **11**, 1047 (1910).

P. Hirsch, Fermentforschung **1**, 33 (1914).

L. H. Adams, Journ. Amer. Chem. Soc. **37**, 1181 (1915).

rays is further quite as in fig. 1. The lower part of the beam leaving lens L, however, does not traverse the vessels, but passes under them through the water in reservoir W, which serves as "Temperier bad" of the vessels. Also these rays of light interfere in O, where they form a fixed image lying below the former, separated from it by a narrow line, and serving as "point de repère" of the zero position. Both images are examined by means of the greatly magnifying cylinder ocular E. The compensator plate P_1 is rotated by means of a lever, which is worked by a micrometer screw with drum D; on this drum a scale is drawn of a hundred divisions. With the instrument used by us one interference band corresponds to 21 scalar divisions. The uncertainty in the adjustment with regard to the coinciding of the two interference images is, after some practice, about half a scalar division.

For the analysis of solutions one has only previously to construct a gauging curve comprising the concentration region used. On the shifting of the central band which seems to take place in this case, compare ADAMS [1]).

3. When it is tried to analyse in this way solutions in organic liquids (for which, of course, the use of so-called "säurefest verschmolzen" vessels consisting entirely of glass, is necessary) the following phenomena are in general observed in the interferometer. The upper interference image is blurred and shifted with regard to the lower one. The bands are permanently oblique and curved, or for a long time. Shaking of the liquid in the vessel (by tapping against the interferometer) indeed promotes the rapidity with which the image is formed, but reproducible results cannot be obtained, and after some time the lines become again shifted and curved.

The causes of these deviations, which render an accurate measurement of course impossible, appeared to be due to the following circumstances:

A. the nature of the bath-liquid.

B. the influence of the temperature on the index of refraction of the solvent;

C. evaporation and distillation against the glass covering-plates of the vessels;

D. absorption of water during the conveying and the staying of the liquid in the vessels.

4. Ad *A. The nature of the bath liquid.*

Compared with by far the majority of the organic liquids the

[1]) ADAMS, Journ. Amer. Chem. Soc. **37**, 1181 (1915).

water used as bath liquid possesses a very small index of refraction. For the substances examined by us we have e.g. at 20°:

tetrachlorethane : $n_D = 1.496$
benzene : $n_D = 1.501$
whereas for water: $n_D = 1.333$

It is easily seen that on use of such solvents with greatly deviating indices of refraction, an exceedingly small departure from the parallel position of the plane-parallel front and back wall of the vessels, brings about a very great displacement of the upper interference image. Let us call the index of refraction of the bath liquid n_1, that of the liquid in the vessel n_2; let further the path in the bath-liquid (a b + c d in fig. 1) passed over by the beam of light which passes C_1, be l_1, that in the vessel l_2. The optical path is then for vessel C_1, the beam passing over the path twice[1]):

$$2\,(n_1 l_1 + n_2 l_2).$$

With perfect parallelism of the plane-parallel plates, this path has the same length for the other beam. If, however, the length of vessel C_2 is e.g. δ more than that of C_1, the optical path is here: $2\,[n_1\,(l_1 - \delta) + n_2\,(l_2 + \delta)]$.

Accordingly the difference in optical path is:

$$\Delta = 2\delta\,(n_2 - n_1),$$

and the displacement of the interference image brought about by this and expressed in bands:

$$N = \frac{\Delta}{\lambda} = \frac{2\delta\,(n_2 - n_1)}{\lambda},$$

if λ represents the wave-length of the light used.

In the case of water and tetrachlorethane $n_2 - n_1$ is $= 0.16$.

From this follows for $\lambda = 0.00058$ mm.:

$N = 550\,\delta$, or expressed in scalar divisions:
$N' = 21 \times 550\,\delta = 11550\,\delta$ scalar divisions.

If the deviation from the parallel position δ is e.g. 0.001 mm., the shifting amounts to no less than $\pm$ 12 scalar divisions. The displacement observed with the vessel used by us appeared to be about 40 scalar divisions. Besides this displacement also the imperfect form of the upper image is partly due to the non-parallelism of front and back wall of the vessels.

5. To obviate this difficulty, a liquid must be chosen as "Tempe-

[1]) We may disregard here the thickness of the plane-parallel plates, the result not being affected by this.

rierbad", whose index of refraction lies as close as possible to that of the liquids to be investigated. Aqueous solutions with strong refractivity (e.g. of cane-sugar or cadmium salts), which are preferable to organic substances, because they do not attack the cement with which the windows are fastened in the reservoir W, proved unsuitable, on account of their high viscosity. After also a number of mixtures of little volatile organic liquids (paraffin-oil-methylsalicylate and methylnonyl-ketone-methylsalicylate) had been tried, in which, however, difficulties also presented themselves, tetrachlorethane itself was taken for it. Previously the rims of the windows had, however, to be protected, for which purpose a layer of an aqueous glue appeared to be very effective, and the paint had to be removed from the reservoir.

6. *Ad B. The influence of the temperature on the index of refraction of the solvent.*

The strong and always recurring curvatures of the bands are partially owing to the great value of the temperature coefficient of the index of refraction in organic liquids, accompanied with a very small specific heat.

From KANONNIKOFF's determinations[1]) $\frac{dn}{dt} = -0.0005$ may e.g. be calculated for tetrachlorethane at $\pm$ 20°.

Benzene has $\frac{dn}{dt} = -0.00065$ at 20°.

A change of temperature of 0.01° gives, therefore, rise to a change in the index of refraction of 6 units in the 6th place of decimals, which when a 2 cm. vessel is used, corresponds to a displacement of 10 scalar divisions. For water these values are about eight times smaller

Exceedingly slight temperature disturbances through addition of heat from outside, which occur especially on a prolonged stay of the observer in the neighbourhood of the apparatus, give much sooner rise to curvatures and displacements of the bands in such liquids than in water.

7. To prevent this:

1. the interferometer was surrounded by a large thermostat, filled with water.

2. a stirrer was placed in the bath liquid.

[1]) Journ. für prakt. Chemie, 32, 520 (1885).

This last improvement at the same time also greatly accelerates the exchange of heat between vessel and bath, so that already after 10 or 15 minutes the reading can take place. To prevent currents in the liquid during the measurement, the stirrer was always stopped a minute beforehand.

8. *Ad C. Evaporation of the liquid in the vessels.*

Only in more concentrated solutions is the error brought about by evaporation, greater than the error of measurement. If e.g. from 2 cc. of a 0.1 N.-solution of $C_2H_2Br_4$ in $C_2H_3Cl_3$ 2 mgr. of $C_2H_2Cl_4$ evaporates, this gives rise in our apparatus to an error of 1.6 scalar divisions. Greasing of the rim of the glass cover can, however, prevent such an evaporation. Another source of errors, however, still continues to exist, viz. distillation against the glass cover. This can often be observed already soon after the filling of the vessel; the liquid which is distilled moves between glass cover and vessel rim on account of surface tension, and attacks the vaseline. To prevent this it is to be recommended [1]) in aqueous solutions to keep the temperature of the "Temperierbad" always some degrees lower than that of the room; in an investigation of more volatile liquids this is, however, not sufficient.

9. It is, however, possible, to avoid this source of error entirely by using for the closure of the vessel, instead of glass plates, massive closing bodies, which occupy the whole vapour space. For this purpose brass blocks were constructed provided with a flat rim, which fit very tightly in the vessel, and leave only a space of ± 2 cc. for the liquid. This is then conveyed to the vessel by the aid of a pipette of 2 cc. capacity, provided with a long capillary passed through a hole bored through the closing block. After the filling the bored hole is closed by a copper ground-in needle. Greasing of the rim of the vessel is now unnecessary; no evaporation takes place through the remaining capillary slits.

10. *Ad D. The influence of the water absorption.*

Dry, organic liquids absorb water vapour from the atmosphere exceedingly rapidly. On account of the great difference in index of refraction between water and those substances, added to the sensitiveness of the method of measurement, however, the presence of exceedingly small quantities of water causes already errors in the

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[1]) Hirsch, Fermentforschung **1**, 38 (1914).

determination which exceed the error of measurement many times. Also when the liquids have not been previously expressly dried with a view to P_2O_5, but when they have only been repeatedly fractionated after preliminary drying with $CaCl_2$ (which was the case with the substances used by us) they show already a strong absorption of water-vapour. Especially for $C_2H_2Cl_4$ this appeared to be the case, but though in a less degree, benzene gave also greatly varying values.

When the determinations are carried out with quite the same precautions as are observed for aqueous solutions, two successive determinations executed directly after each other, already in consequence of this source of error alone, yield greatly deviating values, which e.g. for $C_2H_2Cl_4$ can sometimes differ inter se no less than 40 scalar divisions. In order to ascertain the influence of the water content on the interferometer reading a previously weighed solution of 5 mgr. water in 35 gr. of $C_2H_2Cl_4$ dried on phosphoric acid was measured against this same $C_2H_2Cl_4$. The displacement was 85 scalar divisions. From this follows that the presence of 0.005 mgr. of water in 2 cc. of $C_2H_2Cl_4$ already causes an error of a scalar division.

11. It is self-evident that with such sensitiveness the utmost care should be taken to prevent any contact of the liquid that is to be examined and water-vapour, if an accuracy is to be reached comparable with that in solutions in water. Therefore the liquids were always preserved over phosphoric acid which had been heated at 160° for some days. The storing bottles had the shape as indicated in fig. 3. The conveyance into the vessel took place with the aid of pipettes J, which had been cleaned with benzene, heated, and filled with dry air.

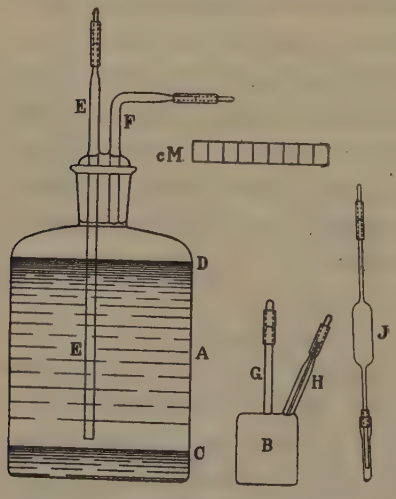

Fig. 3.

These were filled from A by fastening them with a rubber tube on to the tube E, and pressing in dry air at F. The solutions were prepared by weighing in flasks B, which had also been previously rinsed with benzene, heated with evacuation, and filled with dry air. The small quantities of $C_2H_2Cl_4$ were conveyed into them by the aid of a glass capillary. The solvent was directly pressed over from the storing bottle into B, after the tubes G and E had been connected

by means of a siphon-shaped tube. The filling of the pipettes J with the solution from B took also place by pressing in of dry air through tube H, after the capillary of the pipette had been immersed through G in the solution; a rubber ring, put round the capillary, ensured air-tight closure.

When the pipette is closed in the way as is seen in the figure, it is possible to preserve the liquid quite unchanged for 12 hours.

The vessels were always cleaned with benzene instead of alcohol and ether, because the latter causes water-vapour to condense on the glass in case of quick evaporation. Before the filling with liquid the vessels were filled with dry air.

The brass closing blocks also protect the liquid sufficiently against absorption of water-vapour from the atmosphere, so that the interferometer position does not appreciably change during the time taken up by a determination. It is true that in the course of some hours the zero position is slightly displaced in consequence of absorption of water by the pure solvent in vessel C_2; this displacement can, however, easily be determined and taken into account.

12. Only when the precautions described here are observed, is it possible to obtain an accuracy and reproducibility, almost equal to that which is reached for aqueous solutions.

In conclusion we may give a series of measurements referring to solutions of $C_2H_2Br_4$ and $C_2H_2Cl_4$ of different concentrations. With every solution two independent measurements were made (also the zero-position was determined every time anew). The measurements are reduced to vacuum. Just as before a vessel of 2 cm. was used.

concentration in percentages (weighed in)	observed number of scalar divisions		mean	corrected	concentration calculated (in percentages)
	1st measurement	2nd measurement			
0.1256	131.0	129.2	130.1	130.1	0.1262
0.1490	176.3	175.3	175.8	154.8	0.1500
0.2925	346.5	345.1	345.8	303.8	0.2920
0.3208	374.9	375.6	375.3	333.3	0.3199
0.4699	556.9	557.1	557.0	494.0	0.4700
0.5409	655.9	654.9	655.4	571.4	0.5413
0.7166	869.8	869.3	869.6	764.6	0.7166

In this table the concentration is expressed in grammes of $C_2H_2Br_4$ present in 100 gr. of solution. The values given under "corr." are the means, diminished by the correction for the shifting of the central band; it occurs here every time after ± 150 scalar divisions.

Under "calculated" are given the values satisfying the interpolation formula:

$$p = 0.0009772\, n - 0.0000000523\, n^2,$$

which has been calculated from the observations according to the method of least squares. In this p represents the concentration (expressed in percentages), n the corrected number of scalar divisions.

Hence it appears that the reproducibility of the measurements is ± 1 scalar division, corresponding to 0.0009 per cent.

SUMMARY.

The causes of the difficulties met with when it is tried to use the ZEISS waterinterferometer for the analysis of solutions in organic liquids, were discussed and the precautions were mentioned required to carry up the accuracy of the measurements to the same order as can be reached with aqueous solutions.

Utrecht, June 1921. VAN 'T HOFF-*Laboratory*.

Anatomy. — *"Concerning an Isolated Muscle of the Ciliary Body of the Pigeon's Eye, Situated near the Eye-split"*. By J. H. ZALMANN. (Communicated by Prof. J. BOEKE).

(Communicated at the meeting of June 25, 1921).

When making a sagittal section of the pigeon's eye we soon reach, after the cornea and the iris, the eyesplit situated in the basis of the iris and in the basement membrane of the corpus ciliare.

In the ciliary body peripheral to this eyesplit we are then particularly struck with a stout muscular fascicle which, in virtue of its firm structure, projects into the spaces of Fontana and is distinguished from the other ciliary muscles by its peculiar form.

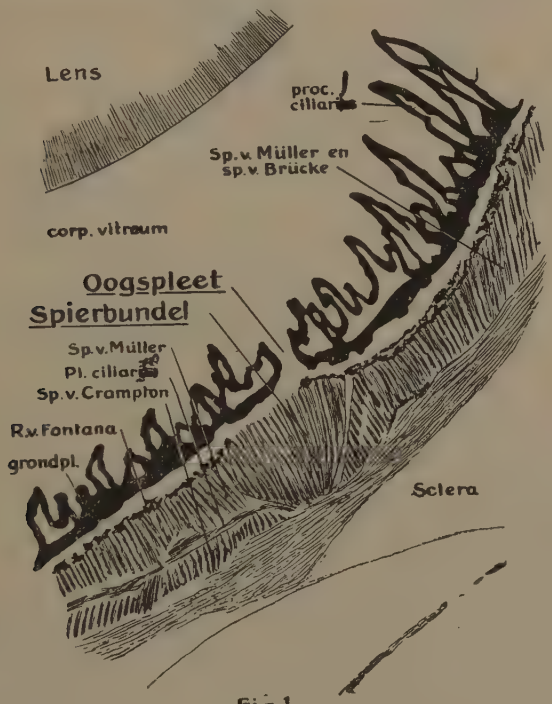

Fig.1.

Sp. v. Müller = Muscle of Müller.
Sp. v. Brücke = „ „ Brücke.
Oogspleet = Eyesplit.
Spierbundel = Muscular fascicle
Sp. v. Crampton = Muscle of Crampton.
R. v. Fontana = Spaces of Fontana.
grondplaat = basement membrane.

For convenience' sake we will term the plane through the beak and the middle of the two pupils: the horizontal plane. The two halves of the bulbus oculi separated by this horizontal plane we will term the lower (nearest to the jaw) and the upper half (nearest to the cranial plane). In the same way the frontal plane through the middle of the two pupils, divides the bulbus into a nasal and a cerebral half.

Now, when we make horizontal sections in the inferior nasal quadrant, we come obliquely upon the above-mentioned eyesplit and muscular fascicle. When making a radial section at an angle of $\pm 45°$ to the horizontal plane, we pass along the muscular fascicle in its whole length, and are thus in a position to determine its anatomic relations.

Besides the method of fixation, embedding in celloidin and the making of sections, there is another, viz. preparing the uvea under the binocular microscope.

To this end the posterior, median, half of the bulbus of a fresh, enucleated eye, was removed.

Along with the anterior stratum of the retina the retinal cell-layer of the processus ciliares was pulled off, in which process also the Zonula of Zinn and the corpus vitrium were removed without injuring the basement membrane of the corpus ciliare. Also the lens, held firmly in its capsule, could now be detached from the processus ciliaris without any harm to the latter.

Now, when we subsequently take up the exposed periphery of the iris and make some cuts in the iris, we can tauten the lig. pectinatum by laying back interiorly — towards the median plane — the sectors formed. With a sharp knife the fibers of this ligament are split close to the basement membrane; then the basement membrane of the corp. cil. is to be laid back still further, the spaces of Fontana are completely open and the medial side of the ciliary muscles is laid bare.

When examining the nasal-inferior quadrant of the urea, before treating it in the manner just described, we observe that the processus ciliares diverge from their radial course at the spot where we should look for the eye-split. They bend round in the direction of the nasal tangent. They make an impression as if they run over an arched sublayer.

Now, when opening in this quadrant the spaces of Fontana, we notice some details, just peripheral to the spot where the processus ciliares bent their course.

At the place of insertion of MÜLLER's muscle into the interior

lamella of the cornea, this muscle is separated from the spaces of Fontana by a pigmented fascia-layer. About halfway this nasal-inferior quadrant this pigmentation is interrupted, and is sharply demarcated from the rest by a pigmented curved line.

When the basement membrane is stretched opposite to this region, a break will be seen in the connecting line between basement membrane and musc. ciliares, formed by the insertion of the tensor chorioideae. The basement membrane bridges the non-pigmented part of the ciliary muscles. (See Fig. 2).

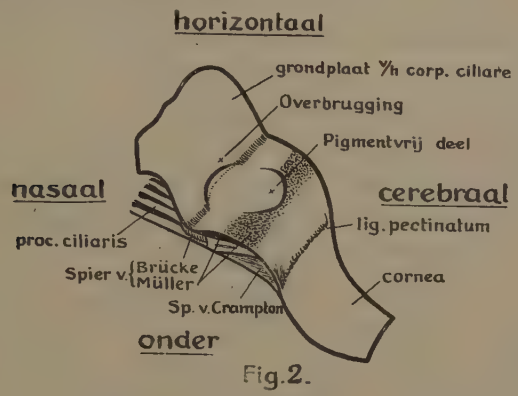

Fig. 2.

Horizontal
grondpl. etc. = basement membr. of the corp ciliare
Overbrugging = Bridging
Pigmentvrij deel = Non-pigmented part.
nasaal = nasal
cerebraal = cerebral
Spier van = Muscle of
onder = Inferior

As is shown in a reconstruction of the sections the form of a non-pigmented spot of the corpus ciliare corresponds with the form of the muscular fascicle. Moreover, the peculair bridging effected by the basement membrane seems to be related to the modified insertion of the M. tensor chorioideae. In Fig. 1 the absence of pigment on the muscular fascicle is also noticeable.

Between the horizontal plane and the part of the corp. ciliare that displays the details alluded to, the ciliary muscles present a regular structure.

From the sclera arise two muscles: CRAMPTON's muscle towards the interior lamella of the cornea and BRÜCKE's muscle as a peripheral part of the tensor chorioideae to the basement membrane of the corp. ciliare. The other part of the tensor chorioideae, MÜLLER's muscle, extends between the inner cornea-lamella and the basement

membrane of the corp. ciliare. The insertion of this muscle into the basement membrane lies slightly more towards the cornea than the insertion of BRÜCKE's muscle. The two parts of the tensor chorioideae are separated from CRAMPTON's muscle by the plexus ciliaris.

When the eyesplit in the basement membrane has been cut into, a muscular fascicle develops in the spaces of Fontana close against MÜLLER's muscle. We now turn away from the horizontal plane and first come upon the place of insertion. The muscular fibers terminate in a tendon, which bends round the compartment of CRAMPTON's muscle, first in conjunction with MÜLLER's muscle and afterwards by itself, and subsequently reaches the inner-lamella of the cornea where the lig. pectinatum takes its origin.

In further sections we see MÜLLER's muscle grow thinner and its tendon elongate in relation to the thinning out of the muscular tissue, ultimately disappearing entirely. The new muscular fascicle has now in part replaced MÜLLER's muscle and partly juts out into the spaces of Fontana.

Hereafter the structure of CRAMPTON's muscle is intensified.

BRÜCKE's muscle shrinks and reduces its place of origin on the sclera, thus making room for the new muscular fascicle. Just where MÜLLER's muscle decreases in size and disappears, this new muscular fascicle imparts twice running a considerable part of its muscular fibers to the basement membrane, which fibers conseqnently perform the function of tensor chorioideae.

The rest, by far the majority of the muscular fibers, have their origin on the sclera, between that of the muscle of BRÜCKE and that of CRAMPTON.

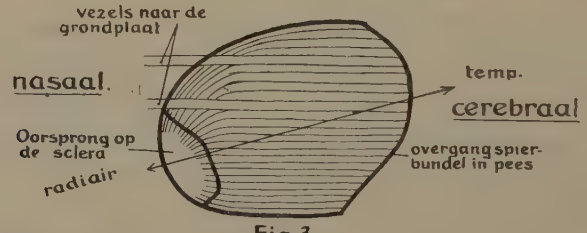

Fig. 3.

Vezels naar de grondplaat = fibers towards the basement membrane.
Nasaal = nasal.
Oorsprong op de sclera = origin of the sclera.
Radiair = radial.
Cerebraal = cerebral.
Overgang spierbundel in pees = transition of muscular fascicle into tendon.

Origin and insertion end approximately in the same radial section, from which it appears that the insertion is much longer than the place of origin. The muscular fascicle is somewhat fan-shaped, diverging from origin to insertion. The muscular fibers remotest from the horizontal plane do not run quite radially, but divert slightly in the direction of the horizontal plane. The course of the muscular fibers nearest to the horizontal plane is initially diverting from the radial direction towards the perpendicular of the horizontal plane. Thereafter they curve in temporal direction, parallel to the remoted muscular fibers. In fig. 1 we also observe a curvature of the muscular fibers and likewise the fan-shape of the muscular fascicle.

Now let us consider the course of the muscular fascicle in a radial plane, vertical to the sclera. The most lateral fibers, — closest to the sclera — proceed linearly from the origin to the insertion. The fibers which help in walling off the spaces of Fontana run in a curve, viz. from the origin first perpendicularly to the sclera, then curving round in the direction of the insertion.

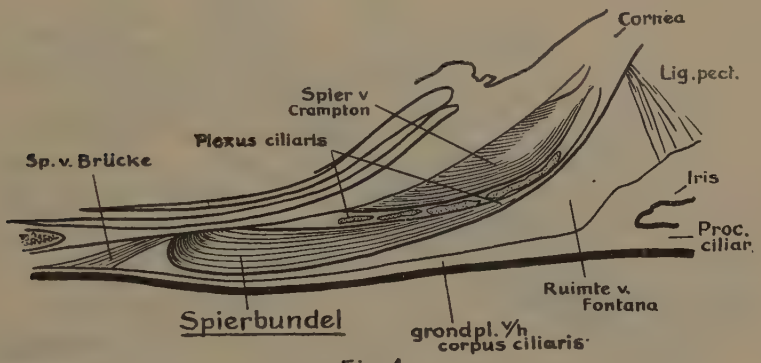

Fig.4.

Spier = muscle.
Spierbundel = muscular fascicle.
Grondvl. v/h = basement membrane of the
Ruimte v = spaces of

Among these curving fibers there are a few which run along a straight line from the place of insertion to the basement membrane. They extricate themselves from the other fibers at the place where the latter change their course, and sometimes in succession where MÜLLER's muscle ceases to exist and the muscular fascicle takes its origin on the sclera.

In fig. 5 the points in the normal corpus ciliare have been marked which in a number of succeeding radial sections have been connected by lines. We here give an explanation of the signs used:

------ the most peripherally located origin of CRAMPTON's muscle.
XXXX insertion and transition into tendon of MÜLLER's muscle.
-·-·-·- Origin and insertion of BRÜCKE's muscle.
——— Boundaries of the muscular fascicle described, with hatched place of origin.

Now let us consider again the structure of the corpus ciliare in sections farther removed from the horizontal plane.

Directly when the origin of the muscular fascicle is left uncut, the available space on the sclera is at once encroached upon by BRÜCKE's muscle. Its origin again pushes on in the direction of the cornea right against that of CRAMPTON's muscle and its muscular fibers push off like a wedge between the sclera and the rest of the muscular fascicle that has been cut into.

Likewise MÜLLER's muscle takes up its old position again. Now, farther and farther away from the horizontal plane the two parts

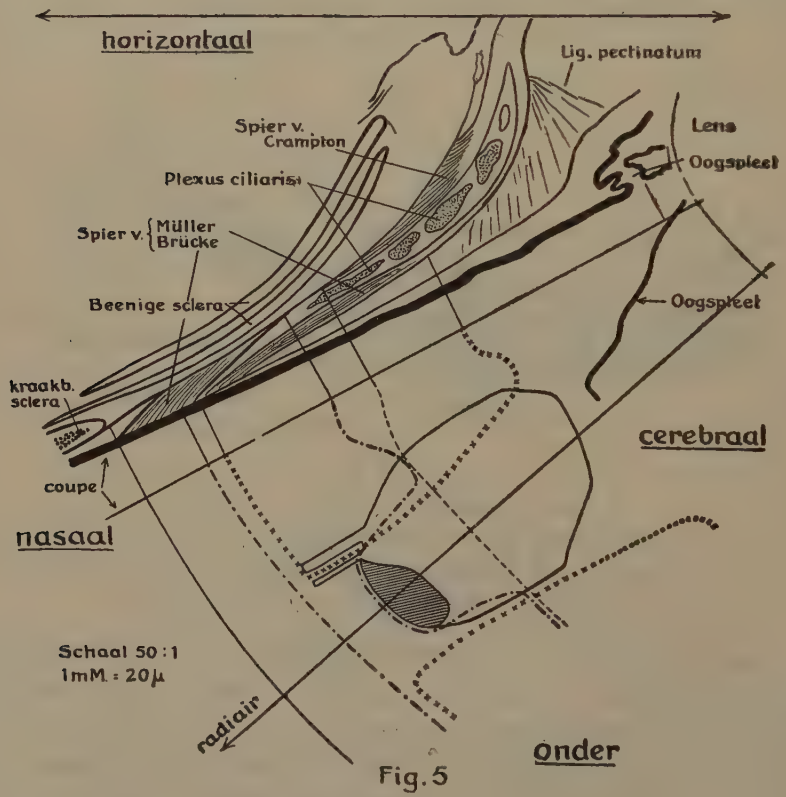

Fig. 5

horizontaal = horizontal
spier van = muscle of
Beenige = osseous
kraakbeenig = cartilaginous
coupe = section

nasaal = nasal
schaal = scale
oogspleet = eyesplit
cerebraal = cerebral
onder = inferior.

of the tensor chorioideae present an alternation in their strongest development. BRÜCKE's muscle and MÜLLER's muscle in turn disappear completely. When BRÜCKE's muscle loses ground CRAMPTON's muscle avails itself of the free space on the sclera to fasten its fibers more backwards.

When summarizing the above we see that in the pigeon's eye there exists near the eyesplit in the nasal-inferior quadrant a muscular fascicle, situated medially to the plexus ciliarus, running from the sclera to the inner lamella-medial part of the cornea. This origin and insertion exclude the muscle from the known types of ciliary muscles of the bird's eye. At that spot CRAMPTON's muscle is strongly developed, BRÜCKE's muscle is only slightly developed, while MÜLLER's muscle has completely disappeared.

In the preparation of the uvea the absence of this muscle accounts as well for the absence of the pigment as for the absence of the bridging by the basement membrane.

The innervation also is furnished by the plexus ciliaris. D. TRETJAKOFF (1906) describes the M. protractor lentis in the salamander's eye. This muscle, like our muscular fascicle lies near the eyesplit in the inferior half of the corpus ciliare. This musc. protactor lentis is not related to the M. tensor chorioideae. TRETJAKOFF's [1]) muscle extends downward from its origin and bends temporally towards the corneo scleral border.

The difference from the discussed muscular fascicle in the pigeon's eye, lies in the fact that contrary to the M. protr. lentis this fascicle extends upwards to bend round temporally afterwards, anyhow as far as those of its fibers are concerned that are nearest to the horizontal plane; also in this that some fibers act like the tensor chorioideae. For the rest there are many points of similarity. Among the eye-split-rests in the deep-sea fishes also a muscle, the M. retractor lentis may be discerned.

This muscular fascicle does not occur in the fowl. NUSSBAUM (1897 [2]) does not mention this pecularity in the corpus ciliare for the simp le reason that he describes the eyesplit in the fowl.

Method of fixation and treatment was as follows. Fixation by means of perfusion of the bloodvessels of the head upwards from the truncus arteriosus cordis and subsequently through submersion into the fixative employed. The fixatives were of low concentration

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[1]) D. TRETJAKOFF 1906 „Der Musc. protract. lentis im Urodelenauge". Anatom. Anzeiger. B. 28.

[2]) M. NUSSBAUM 1897 „Die pars ciliaris des Vogelauges. B. 57, p. 346.

in order to prevent as much as possible dislocation of the retina, viz. formalin 5%, glacial acetic vinegar $1^1/_2$—$^3/_4$ %.

Subsequently the head was deprived of skin, beak, lower-jaw and occiput, frozen on the freezing-microtome and cut through parallel to the horizontal plane as far as the middle of the two pupils. It was then thawed, dehydrated in alcohol, decalcified and embedded in celloidin or parts of it in paraffin. The sections measured 10—30 μ in thickness.

From the Anatomical Institute of Leyden.

Mathematics. — *"On an Integral Notion of* DENJOY." By J. G. VAN DER CORPUT. (Communicated by Prof. ARN. DENJOY).

(Communicated at the meeting of April 30, 1921).

In this paper small Roman letters represent real numbers, Greek letters points in n-dimensional space, R_n, and Roman capitals n-dimensional sets of points; an exception is only made for the letters F, f, Φ and φ, which will always represent functions. The letters a, b, x indicate the products of the coordinates resp. of α, β, ξ. By $\xi\eta$ the point is indicated of which each coordinate is the product of the corresponding coordinates of ξ and η; if E is formed by the points ξ, $E\eta$ represents the set of the points $\xi\eta$. When the coordinates of η are all different from zero, each coordinate of $\frac{\xi}{\eta}$ is equal to the ratio of the corresponding coordinates of ξ and η; $\frac{E}{\eta}$ indicates the set of the points $\frac{\xi}{\eta}$, where ξ is again an arbitrary element of E. When the coordinates of every point ξ of E are different from zero, $\frac{1}{E}$ represents the set of the points $\frac{1}{\xi}$. Finally, when the coordinates of α, β, ξ, η are positive, A, B, X, Y represent the sets of the points of which each coordinate is positive and less than the corresponding coordinate resp. of α, β, ξ, η.

We form a net of cells of n dimensions, i.e. an enumerable sequence of separate, open, connected sets of points $G_i (i = 1, 2, \ldots.)$, measurable (J), so that every point of R_n lies in one of the cells G_i or on its boundary. Of the projection of G_i on each coordinate-axis it is assumed here that when g_i indicates the upper limit, g'_i the lower limit of the distances from the origin to the points of this projection, $\frac{g_i - g'_i}{g'_i}$ approaches to zero at the same time as $\frac{1}{g_i}$. In each cell G_i or on its boundary we choose further a point η_i and we shall indicate the measure of G_i by m_i.

Let $f(\eta)$ be defined for all η in R_n; let p be an arbitrary number between 0 and 1 and let us put

$$\varphi(\xi) = x \sum m_i f(\xi \eta_i)$$

in the points ξ with positive coordinates where the right hand member has meaning; the sum extends over all i for which each coordinate of η_i taken absolutely is more than ρ. As an extension and a slight alteration of the integral-definition (C) given by Prof. A. DENJOY [1]) I shall say that $f(\eta)$ is integrable (C) and that the integral has the value l when the approximate limit of $\varphi(\xi)$ exists and has a value l, independent of the choice of ρ, the net of cells and the points η_i. We write this:

$$\underset{\xi=0}{appr.\ lim.}\ \varphi(\xi) = l;$$

this means that an enumerable set V can be found for which $\underset{\xi=0}{lim.}\ q(\xi) = l$, and the metric density of V at the origin is 1 (here each coordinate of every element of V is considered to be positive). We say that V has a metric density d at the origin, if for each positive number $e < d$ a point α with positive coordinates can be found so that all the points β in A have the property that the measure of the subsets of V in B lies between $(d-e)b$ and $(d+e)b$. Hence $0 \leq d \leq 1$. The following auxiliary theorem is of importance, of which the proof, given by Prof. DENJOY [2]) for linear sets, can be extended at once to more dimensional sets.

Auxiliary proposition 1: *If for every positive* $q < 1$ *a point* α *with positive coordinates and a measurable set* V *can be found for every point* ξ *of which* $|\varphi(\xi) - l| < q$, *while for every point* β *in* A *the subset of* V *in* B *has a measure* $> (1-q)b$, *we have*

$$\underset{\xi=0}{appr.\ lim.}\ \varphi(\xi) = l.$$

For $n = 1$ Prof. DENJOY has found the proof of

Proposition 1: *Every summable function is integrable* (C) *and the integrals are identical.*

We shall precede the proof of this theorem for an arbitrary n by an auxiliary proposition.

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[1]) *Sur l'intégration riemannienne; Comptes Rendus*, 169, (1919); p. 219—221. Prof. DENJOY gives the definition only for $n = 1$. The alteration in the two definitions for $n = 1$ consists chiefly of this, that in the originai definition the length of each interval G_i is supposed to be less than 1, whereas we assume only that this length divided by the distance from G_i to the origin, approaches to zero for $i \to \infty$ Prof. DENJOY gives at the same time two more integral definitions, which he indicates by (A) and (B). Mr. T. J. BOKS studies in his thesis for the doctorate (not yet published) the integral notion (B) and derives the two properties of them, corresponding to the properties 1 and 2 of this paper. Probably this thesis (written at Utrecht) will appear in the Rendiconti di Palermo (1921).

[2]) *Sur les fonctions dérivées sommables, Bulletin de la Société Mathématique de France*, tome 43 (1915); p. 161—248; cf. p. 165—168.

Auxiliary proposition 2: *Suppose* $0 < q < 1$. *If* β *represents a point with positive coordinates, if the non-negative function* $f(\eta)$ *is summable and has a sum* $< q^s$, *the points* ξ *in* B *with* $\varphi(\xi) \geq q$ *form a set with measure* $< hbq$, *where* h *only depends on the choice of* p *and the net of cells.*

Proof: Let R_n' be the part of R_n that has positive coordinates. Let there be associated to each point ξ of R_n' the function $F(\xi)$ indicating the sum of the measures m_i of all the cells G_i of which the point η_i lies in X and each coordinate is more than p. If $F(\xi)$ is positive, each coordinate of ξ is more than p. In accordance with the condition made for the net of cells, we can now determine a number h only dependent on the choice of p and the net of cells, so that the point η the coordinates of which are $\sqrt[n]{h-1}$ times as much as those of ξ, has the property that the cells G_i of which the sum of the measures has been called $F(\xi)$, all lie in Y. Hence $F(\xi) \leq (h-1)x$.

For any point α in B we have

$$(h-1)\int_{R_n'} f(\alpha\xi)\, d\xi \geq \int_{R_n'} F(\xi)\frac{f(\alpha\xi)}{x}\, d\xi = \Sigma' m_i \int_{(\eta_i)} \frac{f(\alpha\xi)}{x}\, d\xi,$$

where the sum has to be extended over all i for which each coordinate of η_i is more than p and where in the last integral each coordinate of ξ exceeds the corresponding coordinate of η_i. The second member is in this case

$$\Sigma' m_i \int_{(\alpha)} \frac{f(\eta_i\xi)}{x}\, d\xi = \int_{(\alpha)} \frac{d\xi}{x}\, \Sigma' m_i f(\eta_i\xi).$$

A similar reasoning gives the analogous expressions for the other parts into which R_n is divided by the coordinate planes and from this there follows through addition

$$(h-1)\int_{R_n} f(\alpha\xi)\, d\xi \geq \int_{(\alpha)} \frac{d\xi}{x}\, \Sigma\, m_i f(\eta_i\xi) = \int_{(\alpha)} \frac{\varphi(\xi)}{x^s} d\xi.$$

From

$$\int_{R_n} f(\alpha\xi)\, d\xi = \frac{1}{a}\int_{R_n} f(\xi)\, d\xi < \frac{q^s}{a}$$

there follows therefore

$$\int_{(\alpha)} \frac{\varphi(\xi)}{x^s}\, d\xi < (h-1)\cdot\frac{q^s}{a}.$$

If $\Phi(\beta)$ and $\Phi(\alpha)$ represent the measures of the sets, formed by the points with the property $\varphi(\xi) \geqq q$ lying resp. in B and A, then $\Phi(\alpha) \leqq a$, and if $B-A$ indicates the set of the points lying in B but not in A, then

$$q(\Phi(\beta) - \Phi(\alpha)) \leqq \int_{B-A} \varphi(\xi)\, d\xi \leqq b^2 \int_{B-A} \frac{\varphi(\xi)}{x^2}\, d\xi$$

$$\leqq b^2 \int_{(\alpha)} \frac{\varphi(\xi)}{x^2}\, d\xi < (h-1)\ \frac{b^2 q^2}{a},$$

hence, if we assume $a = bq$

$$\Phi(\beta) < a + (h-1) \cdot \frac{b^2 q^2}{a} = h\, b\, q.$$

Proof of proposition 1: It is known [1]), that it is possible to find a limited set E with an exterior measure (J) for every number q between 0 and 1 and for every summable function $f(\eta)$, so that $f(\eta)$ is limited and continuous on E and the integral of $|f(\eta)|$ extended over the complement of E, is less than q^2. Let β be an arbitrary point with positive coordinates. If on E we assume $F(\eta) = 0$, outside E, $F(\eta) = |f(\eta)|$ and further

$$\Phi(\xi) = \varkappa \Sigma m_i F(\eta_i \xi),$$

the points ξ in B with the property $\Phi(\xi) \geqq q$ form according to the afore mentioned auxiliary proposition a set with measure $< hbq$.

The points ξ in B with the property $\Phi(\xi) < q$ form therefore a set with measure $> (1-hq)b$.

Let us further assume

$$\overline{\Phi}(\xi) = \varkappa \Sigma m_i f(\eta_i \xi),$$

extended over all i for which $\eta_i \xi$ lies on E. According to the condition made for the net of cells, the dimensions of the cells G_i for which $\eta_i \xi$ lies on the limited set E, approach together with ξ to zero. As on the limited set E with exterior measure (J), $f(\eta)$ is limited and continuous, $f(\eta)$ is integrable on E according to RIEMANN and in accordance with the definition of the integrals of RIEMANN we have in this case

$$\lim_{\xi=0} \overline{\Phi}(\xi) = \int_E f(\eta)\, d\eta.$$

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[1]) Cf. e.g. C. CARATHÉODORY, *Vorlesungen über reelle Funktionen*, 1918, p. 469, proposition 12.

We can therefore find a point α with positive coordinates so that for any point ξ in A

$$|\overline{\Phi}(\xi) - \int_E f(\eta)\, d\eta| < q,$$

hence

$$|\overline{\Phi}(\xi) - \int_{R_n} f(\eta)\, d\eta| < q' + q < 2q.$$

As $|\varphi - \overline{\Phi}| \leqq \Phi$ all points ξ in A for which $\Phi(\xi) < q$, satisfy the inequality

$$|\varphi(\xi) - \int_{R_n} f(\eta)\, d\eta| < 3q,$$

so that any point β in A has the property that the points ξ in B for which this unequality holds good, form a set with measure $> (1 - hq)b$. According to the first auxiliary proposition we have now

$$appr.\ \underset{\xi=0}{lim.}\ \varphi(\xi) = \int_{R_n} f(\eta)\, d\eta,$$

which was to be proved.

A simple application of property 1 is e.g.:

If E indicates a measurable set with finite measure m, $F\left(\frac{E}{\xi}\right)$ the number of points of E with integer coordinates not lying in one of the coordinate planes, we have

$$appr.\ \underset{\xi=0}{lim.}\ \frac{1}{x} F\left(\frac{E}{\xi}\right) = m.$$

We may here replace $F\left(\frac{E}{\xi}\right)$ by all the points on $\frac{E}{\xi}$ with integer coordinates, among others when the intersection of E with each coordinate plane is limited.

It appears from the first property that the integral of DENJOY is at least as general as that of LEBESGUE; from the following proposition (where $n = 1$ is assumed), the correctness will appear of Prof. DENJOY's supposition that the new integral notion is already more general for $n = 1$.

Proposition 2: *If $f(-\xi) = -f(\xi)$, if $f(\xi)$ does not increase with increasing positive ξ, if $f(\xi) = 0$ for $\xi > 1$ and $\underset{\xi=0}{lim.}\ \xi f(\xi) = 0$, $f(\xi - 1)$ is integrable (C) and the integral is* 0.

Two auxiliary propositions precede the proof of this theorem.

Auxiliary proposition 3: *If* $0 < q < 1$ *and the segment* S_j $(j = 1, 2, \ldots)$ *does not contain the origin, has one point in common with the segment* $\delta_j\,\delta_{j-1}$ *and has a length* $< q\,(\delta_{j-1} - \delta_j)$, *where* $\delta_{j-1} > \delta_j > 0$ *and* $\lim_{j=\infty} \frac{\delta_{j-1} - \delta_j}{\delta_{j-1}} = 0$, *the set formed by the segments* S_j *has at the origin a metric density* $\leqq q$.

Proof: We shall assume that for $j \to \infty$ δ_j approaches the origin because else the auxiliary proposition would be evident. Let ξ be an arbitrary point to the right of the origin and let u be the smallest value of j for which S_j contains the point ξ or a point to the left of it. Then

$$\delta_{u-1} = \delta_u + (\delta_{u-1} - \delta_u) \leqq \xi + \text{length } S_u + (\delta_{u-1} - \delta_u),$$

hence

$$\delta_{u-1} \leqq \xi + (1 + q)\,(\delta_{u-1} - \delta_u) \quad \ldots \ldots \quad (1)$$

The subset of the segments S_j lying between the origin and ξ, has therefore a measure which after division by ξ is not more than

$$q\,\frac{\delta_{u-1}}{\xi} \leqq q + q(1 + q)\,\frac{\delta_{u-1} - \delta_u}{\xi} \quad \ldots \ldots \quad (2)$$

When ξ approaches the origin, u increases unlimitedly; $\frac{\delta_{u-1} - \delta_u}{\delta_{u-1}}$ approaches therefore to zero and according to (1) the lower limit of $\frac{\xi}{\delta_{u-1}}$ is not less than 1. The last term of (2) approaches to 0, so that the metric density of the set formed by the segments is at the origin $\leqq q$.

Auxiliary proposition 4: *When* $\lim_{\xi=0} \xi f(\xi) = 0$ *and when to any point to the right of the origin a number* i *is conjugated, so that* $\frac{1}{\xi}$ *lies in* G_i *or on its boundary, we have*

$$\text{appr. } \lim_{\xi=0} \xi\, m_i f(\eta_i \xi - 1) = 0.$$

Proof: For $\xi \to 0$, $i \to \infty$, hence according to the condition imposed on the net of cells

$$\eta_i \xi - 1 = \frac{\eta_i - \frac{1}{\xi}}{\frac{1}{\xi}} \to 0, \quad \frac{m_i \xi}{\eta_i \text{ measure } \frac{1}{G_i}} \to 1, \quad (\eta_i \xi - 1) f(\eta_i \xi - 1) \to 0. \quad (3)$$

Suppose $0 < q < 1$; let E be the set of the points in $\frac{1}{G_i}$ $(i = 1, 2, \ldots,$

omitting the value or values of i for which the origin lies in G_i or on its boundary) the distances of which to $\frac{1}{\eta_i}$ are more than $\frac{1}{2}q$ measure $\frac{1}{G_i}$. According to the preceding auxiliary proposition (where the points σ_j indicate the endpoints of the sets of points $\frac{1}{G_i}$, if necessary after reversal of the direction of the ξ-axis), the complement of E has at the origin a metric density $\leqq q$, hence E itself has a metric density $\geq (1-q)$. For any ξ on E we have $\left|\xi - \frac{1}{\eta_i}\right| \geqq \frac{q}{2}$ measure $\frac{1}{G_i}$, hence according to (3)

$$\xi m_i f(\eta_i \xi - 1) = \frac{m_i \xi}{\eta_i \text{ measure} \frac{1}{G_i}} \cdot \frac{\text{measure} \frac{1}{G_i}}{\xi - \frac{1}{\eta_i}} \cdot (\eta_i \xi - 1) f(\eta_i \xi - 1) \to 0.$$

As this holds good for any q between 0 and 1 the theorem in question follows from this in connection with the auxiliary proposition 1.

Proof of proposition 2: The relation to be proved is:

$$\underset{\xi=0}{appr.\ lim.}\ \xi \Sigma m_i f(\eta_i \xi - 1) = 0 \quad \ldots\ldots \quad (4)$$

As f is zero to the left of the point -1, i takes in this sum only values for which the whole or part of G_i lies to the right of the origin. If ω is an arbitrary point to the right of the origin, every term in (4) with such i that the whole or part of G_i lies to the left of ω, approaches to zero with ξ and the number of these terms is limited, so that in (4) we need only take into account those values of i for which the cells of G_i lie to the right of ω. We shall indicate those cells, arranged from left to right, by $\theta_{j-1}\theta_j$ $(j = 1, 2, \ldots)$. As $\lim_{j=\infty} \frac{\theta_j}{\theta_{j-1}} = 1$, we can choose ω in such a way that always $\frac{\theta_j}{\theta_{j-1}} < 2$. According to the preceding auxiliary proposition, for $\xi \to 0$ the approximate limit of the term or the terms in (4) with such i that $\frac{1}{\xi}$ lies in G_i or on its boundary, is zero. We can therefore skip this value or these values of i and we shall indicate this by an accent to Σ, so that it is sufficient to prove

$$\underset{\xi=0}{appr.\ lim.}\ \Phi(\xi) = 0, \quad \text{where} \quad \Phi(\xi) = \xi \sum_{j=1}^{\infty}{}' m_j f(\eta_j \xi - 1).$$

With a view to this we shall first prove

$$\underset{\xi=0}{appr.\,lim.}\, F(\xi) = 0, \quad \text{where} \quad F(\xi) = \xi \sum_{j=1}^{\infty}{}' m_j \{f(\theta_{j-1}\xi - 1) - f(\theta_j \xi - 1)\}.$$

As $f(\xi)$ does not increase for increasing ξ when it does not pass the origin, the terms of the latter sum are $\geqq 0$.

If q represents an arbitrary number between 0 and $\frac{1}{2}$, $\frac{1}{\theta_j} = \delta_j$ and $l_j =$ the length of the segment (σ_j, σ_{j-1}), $\frac{\sigma_{j-1}}{\sigma_j} < 2$ hence $q l_j < l_j = \left(\frac{\delta_{j-1}}{\delta_j} - 1\right)\delta_j < \delta_j$, so that if each of the segments (σ_j, σ_{j-1}) is produced at both ends with an interval of length $q l_j$, none of these productions contains the origin. According to the auxiliary proposition 2 the metric density of the set formed by these productions, is at the origin at most $2q$. Now let β be an arbitrary point to the right of the origin and let E be an arbitrary measurable set to the right of β which has no point in common with any of these productions. Let us first consider the terms in $F(\xi)$ for which $\xi \geqq \sigma_j$; the accent excludes the case $\delta_j \leqq \xi \leqq \sigma_{j-1}$ and the production $\sigma_{j-1} < \xi < \sigma_{j-1} + q l_j$ does not occur in E, hence

$$\int_E \frac{f(\theta_{j-1}\xi - 1) - f(\theta_j \xi - 1)}{\xi} d\xi \leqq \int_{\delta_{j-1}+q l_j}^{\infty} = \int_{q\theta_{j-1} l_j}^{\infty} \frac{f(\xi)}{1+\xi} d\xi - \int_{(1+q)\theta_j l_j}^{\infty} = \int_{q\theta_{j-1} l_j}^{(1+q)\theta_j l_j} \leqq \int_{q\theta_{j-1} l_j}^{2\theta_j l_j} f(\xi) d\xi. \quad (5)$$

In the remaining terms of $F(\xi)$, $\xi < \sigma_j$, hence for these, because the production $\sigma_j - q l_j < \xi < \sigma_j$ does not occur in E, we have

$$\left.\begin{aligned} \int_E \frac{f(1-\theta_j\xi) - f(1-\theta_{j-1}\xi)}{\xi} d\xi \leqq \int_{\beta}^{\delta_j - q l_j} = \int_{1-\theta_{j-1}\beta}^{(1+q)\theta_{j-1}} \frac{f(\xi)}{1-\xi} d\xi - \int_{1-\theta_j\beta}^{q\theta_j l_j} = \\ = \int_{q\theta_j l_j}^{(1+q)\theta_{j-1} l_j} - \int_{1-\theta_j\beta}^{1-\theta_{j-1}\beta} \leqq \int_{q\theta_j l_j}^{(1+q)\theta_{j-1} l_j} \leqq 4 \int_{q\theta_{1-j} l_j}^{2\theta_j l_j} f(\xi)\, d\xi, \end{aligned}\right\} \quad (6)$$

for from $\xi \leqq (1+q)\,\theta_{j-1} l_j = (1+q)\left(1 - \frac{\theta_{j-1}}{\theta_j}\right) < \frac{3}{4}$ there follows $1 - \xi > \frac{1}{4}$.

As $\underset{j=\infty}{lim}\, \theta_j l_j = 0$ and $\underset{\xi=0}{lim}\, \xi f(\xi) = 0$ we can choose v_j (only dependent on the function f and the net of cells) in such a way

that $\lim_{j=\infty} v_j = 0$ and that the inequality $f(\xi) < \frac{v_j}{5\xi}$ follows from $0 < \xi \leqq 2\,\theta_j\, l_j$.

In this case

$$\int_{q\theta_{j-1}l_j}^{2\theta_j l_j} f(\xi)\, d\xi < \frac{v_j}{5} \log \frac{2\theta_j}{q\theta_{j-1}} < \frac{v_j}{5} \log \frac{4}{q}.$$

If therefore u indicates the greatest value of j for which $\theta_{j-1}\beta - 1 \leqq 1$ we have according to (5) and (6)

$$\int_E \frac{F(\xi)}{\xi^2}\, d\xi < \log \frac{4}{q} \sum_{j=1}^{u} v_j\,(\theta_j - \theta_{j-1}), \quad . \quad . \quad . \quad . \quad . \quad (7)$$

because $f(\theta_{j-1}\,\xi - 1)$ and $f(\theta_j\,\xi - 1)$ are zero for $j > u$ owing to $\xi \geqq \beta$. The aforesaid inequality holds for any measurable set E to the right of β which has no point in common with any of the productions of the segments (σ_j, σ_{j-1}). We shall now assume that this set E lies to the left of a point γ and that for any point ξ of E the inequality $F(\xi) \geqq q$ holds good. In this case the left-hand member of (7) is not less than $\frac{q}{\gamma^2}$ multiplied by the measure of E. Now we can choose γ so close to the origin that the measure of the subset between 0 and γ of the productions is $< 3q\gamma$. The set between 0 and γ of the points ξ for which $F(\xi) \geqq q$, has therefore a measure less than

$$3q\gamma + \beta + \text{measure } E < 3q\gamma + \beta + \frac{\gamma^2}{q} \log \frac{4}{q} \sum_{j=1}^{u} v_j\,(\theta_j - \theta_{j-1}) \quad . \quad (8)$$

For $q < \frac{1}{2}$ the right-hand member is less than $6q\gamma$, if γ lies close enough to the origin and β is chosen properly. With a view to this we assume $\beta = q\gamma$; if γ lies close enough to the origin, u is so great that for any $j > u$

$$v_j < \frac{q^2}{4 \log \frac{4}{q}};$$

In this case we can therefore define an integer number $w \leqq u$ independent of γ, so that this inequality holds good for any $j > w$. Finally we choose γ so close to the origin that

$$\frac{\gamma}{q} \log \frac{4}{q} \sum_{j=1}^{w} v_j\,(\theta_j - \theta_{j-1}) < q$$

In this case:

$$\frac{\gamma^2}{q}\log\frac{4}{q}\sum_{u\geq j>w} v_j(\theta_j-\theta_{j-1})<\frac{1}{4}\gamma^3q^2\sum_{u\geq j>w}(\theta_j-\theta_{j-1})$$

$$\leq\frac{1}{4}\gamma^3q^2\theta_u<\tfrac{1}{2}\gamma^3q^2\theta_{u-1}<\gamma^3q^2\cdot\frac{1}{\beta}=\gamma q.$$

The second member of (8) is then $<(3+1+1+1)q\gamma=6q\gamma$, so that the set of the points ξ between 0 and γ for which $F(\xi)\geq q$, has at the origin a measure $<6q\gamma$. The points ξ between 0 and γ with $F(\xi)<q$ form therefore a set with measure $>(1-6q)\gamma$. This holds good for any q between 0 and $\frac{1}{2}$; according to the first auxiliary proposition the non-negative function $F(\xi)$ has in this case for $\xi\to 0$ an approximate limit 0. By the aid of this we shall prove that also $\Phi(\xi)$ has an approximate limit 0 for $\xi\to 0$. From the aforesaid,

$$f(\theta_{j-1}\xi-1)\geq f(\eta_j\xi-1)\geq f(\theta_j-1)$$

and

$$m_j f(\theta_{j-1}\xi-1)\geq\int_{G_j} f(\eta\xi-1)\,d\eta\geq m_j f(\theta_j\xi-1)$$

follows

$$\text{appr. lim.}_{\xi=0}\left\{\Phi(\xi)-\xi\sum_{j=1}^{\infty}\int_{G_j} f(\eta\xi-1)\,d\eta\right\}=0. \qquad (9)$$

The values ξ for which $\frac{1}{\xi}$ lies on the boundary of one of the cells G_j, form an enumerable set so that we need only consider the case that ξ lies inside a cell G_i. If λ_i indicates the left extremity, ϱ_i the right extremity of this cell, we have owing to $f(-\xi)=-f(\xi)$

$$\xi\sum_{j=1}^{\infty}\int_{G_j} f(\eta\xi-1)\,d\eta=\xi\int_{-\infty}^{\lambda_i} f(\eta\xi-1)\,d\eta+\xi\int_{\varrho_i}^{\infty} f(\eta\xi-1)\,d\eta=$$

$$=\int_{-\infty}^{\lambda_i\xi-1} f(\eta)\,d\eta+\int_{\varrho_i\xi-1}^{\infty} f(\eta)\,d\eta=\int_{\varrho_i\xi-1}^{1-\lambda_i\xi} f(\eta)\,d\eta.$$

The difference of the positive numbers $1-\lambda_i\xi$ and $\varrho_i\xi-1$ is less than their sum $\xi(\varrho_i-\lambda_i)$; $f(\eta)$ is absolutely taken $\leq f(\varrho_i\xi-1)+f(1-\lambda_i\xi)=f(\varrho_i\xi-1)-f(1-\lambda_i\xi)$, hence the absolute value of the latter integral is less than $\xi(\varrho_i-\lambda_i)\{f(\varrho_i\xi-1)-f(\lambda_i\xi-1)\}$ and

the approximate limit of this is according to the auxiliary proposition 3, equal to zero. As appears from (9) also the approximate limit of $\Phi(\xi)$ is in this case equal to zero for $\xi \to 0$ and herewith proposition 2 has been proved.

This property gives measurable functions that are integrable (C) but not summable. These functions are all unlimited at the upper side as well as at the lower side. That this is also necessary appears from

Proposition 3: *If a measurable function* $f(\xi)$ *is integrable* (C) *and limited at the upper or at the lower side, this function is summable.*

Proof: Assume for instance that $f(\xi)$ is limited at the lower side; let us put $f_t(\xi) = f(\xi)$ or $= t$ according to whether $f(\xi) \leqq$ or $> t$. The limited function $f_t(\xi)$ is summable, hence integrable (C), and the integrals are equal. Let this common value be called s_t. For increasing t s_t does not decrease and from $f_t(\xi) \leqq f(\xi)$ follows that s_t is not more than the integral (C) of $f(\xi)$. The integrals s_t are therefore limited, hence $f(\xi)$ is summable.

Some properties holding good for the integrals of LEBESGUE, remain valid, others do not. From proposition 2, for instance, appears that the following two properties of the integrals of LEBESGUE are lost: When a function is summable, its absolute value is also summable. When a function is summable in a set, it is also summable in any measurable subset.

Finally we shall discuss three more properties that remain valid.

Proposition 4: *When a function* $f(\xi)$ *is integrable* (C), *the points for which* $f(\xi)$ *is infinite, form a set with measure zero.*

Proof: We shall show that a function with the property that the points ξ for which the coordinates are positive and $f(\xi)$ is infinite, form a set E with positive exterior measure, is not integrable (C); we can confine ourselves to a very simple net of cells, namely to the net of cells of which every cell consists of an n-dimensional cube of which the sides have a length 1 and are parallel to the coordinate-axes, and of which the centre coincides with a point with integer coordinates. In an analogous way each of the $2^n - 1$ other parts into which R_n is divided by the coordinate-axes, may be treated.

The exterior measure of E being positive, there exists a beam H of which the coordinates of the angular points are positive and the sides are parallel to the coordinate-axes, while a subset D of E in H at a positive distance from the boundary, has a positive exterior measure. By enlarging the beam, if necessary, we can attain that if α and β represent the angular points of H, resp. with the least and with the largest coordinates, the coordinates of β are twice those

of a. Now let the exterior measure of D be w times the volume of the beam H. If η describes the sequence of points of which the coordinates are powers of 2 with integer non-negative exponents, the beams $\frac{H}{\eta}$ occupy together exactly the set of points B. The set $\frac{D}{\eta}$ lies in $\frac{H}{\eta}$ at a positive distance from the boundary and the exterior measure of $\frac{D}{\eta}$ is w times the volume of $\frac{H}{\eta}$. The exterior measure of the set formed by all $\frac{D}{\eta}$ is therefore wb. Any measurable set containing all the sets $\frac{D}{\eta}$ has accordingly at the origin a metric density $\geqq w$. In each of the sets $\frac{D}{\eta}$, $\varphi(\xi)$ contains at least one term which is infinite, so that $\varphi(\xi)$ is there infinite or indeterminate; any measurable set where $\varphi(\xi)$ has a definite finite value, has therefore at the origin a metric density $\leqq 1-w$, so that $f(\xi)$ is not integrable (C).

Proposition 5: *If* $f_1(\xi)$ *and* $f_2(\xi)$ *are integrable* (C) *and* e_1 *and* e_2 *represent two arbitrary finite numbers, a function coinciding with* $e_1 f_1(\xi) + e_2 f_2(\xi)$ *when this expression has meaning, is integrable* (C) *and the integral is* e_1 *times the integral of* $f_1(\xi)$, *augmented by* e_2 *times the integral of* $f_2(\xi)$.

Proof: Two functions are called equivalent if they coincide except perhaps in a set with measure zero. It appears from the preceding proposition that any function which is integrable (C), is equivalent to a finite function, from the first proposition that two equivalent functions are either both or neither integrable (C). As the above mentioned proposition for finite functions follows immediately from the definition of the integral notion (C), the property holds generally, because, if necessary, the functions can first be replaced by the equivalent finite functions.

Proposition 6: *For a monotone series of measurable* (C) *functions* $f_t(\xi)$ $(t = 1, 2, \ldots)$ *approaching to* $f(\xi)$, *the series of the integrals is likewise monotone. Further the limit function is integrable* (C) *only when the series of the integrals is limited. If this is the case, the integral of* $f(\xi)$ *is equal to the limit for* $t = \infty$ *of the integral of* $f_t(\xi)$.

Proof: If $f(\xi)$ is integrable (C), the integral is not less, resp. not more, than that of $f_t(\xi)$, so that the non-decreasing, resp. non-

increasing, series of integrals is limited. Now let s_t be the integral (C) of $f_t(\xi)$ and $s = lim\, s_t$. The function $f_t(\xi) - f_1(\xi)$ is either always $\geq$ or always ≤ 0 and integrable (C), hence according to proposition 3 it is summable with $s_t - s_1$ for sum. As the monotone series $s_t - s_1$ has $s - s_1$ for limit, $f(\xi) - f_1(\xi)$ is summable with $s - s_1$ for sum. From this it ensues that $f(\xi) - f_1(\xi)$ is integrable (C) with $s - s_1$ for integral, so that $f(\xi)$ is integrable (C) with s for integral.